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A simple extension of a von Neumann regular ring

By

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In this note all rings are commutative rings with units. Let R be a (von Neumann) regular ring (i.e. an absolutely flat ring in Bourbaki's sense) and let $R[\alpha]$ be a reduced simple extension of R. Let $R[x]$ be a polynomial ring of a variable x over R, I the kernel of the canonical homomorphism T of $R[x]$ onto $R[\alpha]$ such that $T(x)=\alpha$.

In this note, first we shall give some conditions on I for $R[\alpha]$ to be quasi-regular. (Following [3], we say that a ring is quasi-regular if its total quotient ring is regular.) And then we shall give some results relating to the dondition (F) in [1]. Throughout this paper, we use the above notation. In particular, I is a semi-prime ideal of $R[x]$ with $I \cap R=(0)$.

We begin with an easy lemma.

Lemma 1. *Every maximal ideal of* $R[x]$ *is of height* 1 *and contains unique minimal prime ideal of the form* $\mathfrak{m}R[x]$ *where* $\mathfrak{m}$ *is a maximal ideal of* R.

The proof is easy and we omit it.

Corollary 2. *Every localization of* $R[\alpha]$ *at a maximal ideal is an integral domain.*

Proposition 3. $R[\alpha]$ *is quasi-regular if and only if* $R[\alpha]$ *is a p.p. ring, that is, every principal ideal of* $R[\alpha]$ *is projective as an* $R[\alpha]$*-module.*

Proof. This follows from Corollary 2 and [2].

Remark 4. In the proof of Proposition 1 in [2], it is shown that if a ring R' is a p.p. ring, then every idempotent in $Q(R')$ (=total quotient ring of R') is contained in R'.

For an $f(x) \in R[x]$, we denote by $c(f)$ the ideal of R generated by coefficients of $f(x)$. For an ideal J of $R[x]$ we denote by $C(J)$ the ideal of R generated by $\{c(f) \mid f(x) \in J\}$.

Theorem 5. *Assume that* $C(I)=R$. *Then* $R[\alpha]$ *is regular and is integral over*