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A generalization of Riemann-Roch theorem and certain algebra of meromorphic functions on symmetric Riemann surfaces

Dedicated to Professor Yukio Kusunoki on his sixtieth birthday

By

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Introduction

The theory of abelian integrals on arbitrary open Riemann surfaces has been attempted in several ways by putting restrictions on the boundary behavior of meromorphic functions and differentials under consideration (see references). In this direction, Kusunoki [6] introduced the notion of canonical potentials and gave a formulation of Riemann-Roch's theorem and Abel's theorem, which deal with the class of meromorphic functions whose real parts are canonical potentials (cf. [5], [7]). By using the notion of behavior spaces, Yoshida [21], Shiba [17] and others showed the extended theory corresponding to various classes of meromorphic functions. However the extended theory is yet limited in the point that the differentials used in the argument of behavior spaces are assumed to be semiexact. Further, in contrast with the classical theory, those classes are real vector spaces and the multiplication of two meromorphic functions in the concerned class does not always belong to that class. In order to improve these points we shall show in this paper a generalized Riemann-Roch theorem by using certain new behavior spaces over the complex number field with less restrictions. That is, we leave out the period conditions in Shiba's behavior spaces, which are required to make use of Riemann's period relation in proving the theory, and under the present conditions we are able to prove a Riemann-Roch theorem and an Abel's theorem without direct use of Riemann's period relation. To show a typical application of these theorems we introduce the symmetric behavior space on symmetric Riemann surfaces which was considered by Matsui [9] from a different point of view. Meromorphic functions subject to the symmetric behavior space are not only Dirichlet bounded in a neighbourhood of the ideal boundary by definition, but also bounded over there. Further, the concerned class of meromorphic functions in our Riemann-Roch theorem is closed by multiplications. At last some simple examples of meromorphic functions with symmetric behavior will be shown.