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# On Teichmüller spaces and modular transformations

Dedicated to Professor Yukio Kusunoki on his 60th birthday

By

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## § 1. Introduction

Let $G$ be a finitely generated Fuchsian group of the first kind acting on the upper half plane $U$ such that the Riemann surface $U/G$ is of type $(p, n)$. The Teichmüller space $T(G)$ of $G$ is identified with a bounded domain in $C^{3p-3+n}$, which is called the Bers embedding of $T(G)$ (Bers [5]). Recently, Bers investigated the action of modular transformations on the boundary $\partial T(G)$ of $T(G)$ in the Bers embedding (cf. [8], [9]). He showed that all modular transformations can be extended to some set of boundary points which is dense in $\partial T(G)$ and that the infinite iterations of a hyperbolic modular transformation accumulate to boundary points corresponding to totally degenerate groups.

In this paper, we shall investigate the infinite iterations of parabolic and pseudo-hyperbolic modular transformations. Furthermore, we shall give a new characterization of the Thurston-Bers classifications of modular transformations in terms of their infinite iterations. Roughly speaking, accumulation points of elliptic, parabolic, pseudo-hyperbolic and hyperbolic modular transformations correspond to quasi-Fuchsian groups, regular $b$-groups, degenerate cusps and totally degenerate groups, respectively (Theorem 3.3). And we shall give some examples about the infinite iterations of pseudo-hyperbolic modular transformations (§4).

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## § 2. Preliminaries

In this section, we shall introduce some notations and recall some known results (for details see Bers [5], [7] and Kra [10]).

Throughout this paper, we denote by $G$ a finitely generated Fuchsian group of the first kind acting on the upper half plane $U$ such that $U/G$ is a Riemann surface of type $(p, n)$ with $2p+n-2>0$, and denote by $\pi$ a canonical projection of $U$ onto $U/G$.