J. Math. Kyoto Univ. (JMKYAZ)
25-4 (1985) 597-617

Square integrable harmonic differentials on arbitrary Riemann surfaces with a finite number of nodes

Dedicated to Professor Yukio Kusunoki on his sixtieth birthday

By

Masahiko TANIGUCHI

(Received November 21, 1983)

Introduction

A quasiconformal mapping between two Riemann surfaces induces an isomorphism between the Hilbert spaces of square integrable harmonic differentials on those surfaces, and it is known (cf. [10]) that such an isomorphism preserves several important subspaces. The first purpose of this paper is to generalize such an isomorphism for the case of a deformation from a Riemann surface with a finite number of nodes to another (cf. §1-1°)). Let $(f; R, R_0)$ be an allowable deformation (cf. §2-1°)) from such a surface R to another R_0, then we will show in §2 (Theorem 1) that the mapping H_f naturally induced from f is a bounded linear injection from the Hilbert space $\Gamma_h(R_0)$ of square integrable harmonic differentials on R_0 into $\Gamma_h(R)$, and has similar properties as in the case of quasiconformal mappings. We also give an estimate of the norm of H_f which is coincident with the known one when f is a quasiconformal mapping.

Now there are several investigations concerning on continuity properties of the above Marden-Minda's isomorphisms on the Teichmüller space (cf., for example, [7] and [12]). The second purpose of this paper is to show certain continuity property of H_f on the finitely augmented Teichmüller space $\hat{T}(R^*)$ of arbitrary Riemann surface R^* (cf. §1-1°)). Actually, we will show in §3 (Theorem 4) that $H_{f_k}(\omega)$ converges to ω strongly metrically for every $\omega \in \Gamma_h(R_0)$ when (R_k corresponding to f_k converges to R_0 and) $\{f_k\}_{k=1}$ is an admissible sequence.

Also we state related results on Dirichlet finite harmonic functions. See Theorem 2 in §2-2°) and Theorems 5 and 6 in §3-2°).

§1 is preliminaries, where we give definitions of notions concerning on the finitely augmented Teichmüller space and the spaces of differentials and functions. Theorem 1 is proved in §2-3°), and a general sufficient condition with which a given sequence of differentials converges strongly metrically is given in §3-1°)