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Some deformations of codimension two discrete groups

Dedicated to Yukio Kusunoki on his sixtieth birthday

By

Bernard MASKIT*

1. Let H^n be hyperbolic n-space, and let $\tilde{L}^n$ denote the group of isometries of H^n; the orientation preserving half of $\tilde{L}^n$ is denoted by L^n. A parabolic element of L^n is a transformation which has exactly one fixed point in the Euclidean closure of H^n. If we normalize so that H^n is the upper half-space, and the parabolic element j has its fixed point at ∞, then, in its action on $\hat{E}^{n-1}=E^{n-1}\cup\{\infty\}=\partial H^n$, $j(x)=r(x)+b$, where r is an orthogonal transformation, and b is not in the range of $1-r$. If $r=1$, then j is a *pure* parabolic transformation, or translation, while if $r\neq 1$, j is *impure*.

The following question was posed by John Morgan (oral communication). Is there a cofinite volume discrete subgroup G of L^n, containing pure parabolic elements, and a deformation $\tilde{G}$ of G in some L^m, $m>n$, where the corresponding parabolic elements of $\tilde{G}$ are impure?

Note that if $m<4$, then every parabolic element of L^m is pure.

For $m=n+1$, it follows from a theorem of Cheeger and Gromoll [1] that a discrete free abelian group of orientation preserving Euclidean motions of rank $n-1$, acting on E^n, contains only pure parabolic elements; we outline an elementary proof of this fact below. It follows that if G is a discrete subgroup of L^n, of cofinite volume, and G contains only pure parabolic transformations, then no deformation of G in L^{n+1} contains impure parabolic elements.

For higher codimension, we give examples to show that one can deform a pure parabolic subgroup into an impure one. These examples also serve as examples for the following. For every $n\geq 4$, and for every positive $k\leq n-3$, there is a family of non-conjugate discrete subgroups $\{G_\alpha\}$ of $\tilde{L}^n$, with the following properties. The family is parametrized by $(S^1)^k$; for $n=4$ and 5 the G_α all have the same limit set, a Euclidean sphere of dimension k; for all $n\geq 4$ and for almost all α, the stabilizer in G_α of the hyperbolic $(k+1)$-plane spanning the limit set is the identity (in particular, for almost all α, G_α is not conjugate in $\tilde{L}^n$ to a subgroup of $\tilde{L}^{k+1}$); and all the G_α have the same finite sided fundamental polyhedron, with the same combinatorial identifications.

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