J. Math. Kyoto Univ. (JMKYAZ)
26-2 (1986) 323-331

The incompressible limit and the initial layer of the compressible Euler equation

By

Seiji UKAI

1. Introduction.

The Euler equation of compressible ideal fluid flow in $\boldsymbol{R}^n$ is written, in appropriate nondimensional form (cf. [5]), as

$$\begin{aligned} &\frac{1}{\gamma p}(p_t+v\cdot\nabla p)+\nabla\cdot v=0, \\ (1.1)\qquad &\rho(v_t+v\cdot\nabla v)+\lambda^2\nabla p=0, \\ &(p, v)|_{t=0}=(p_0, v_0). \end{aligned}$$

Here, the unknowns are the pressure $p=p(t, x)>0$ and velocity $v=v(t, x)\in\boldsymbol{R}^n$, $t\geqq 0$, $x\in\boldsymbol{R}^n$, while ρ is the density governed by the equation of state $p=\rho^{\gamma}$, $\gamma>1$, and λ, the parameter arising from nondimensionalization, is $M^{-1}\gamma^{-1/2}$, M being the Mach number.

In this paper we discuss the limit of solutions as $\lambda\to\infty$. Some fundamental facts on this limit have been established by Klainerman and Majda in [5], (see also [4] for the periodic case and [1], [2] for bounded domains). In particular, it is shown that unique solutions exist for all large λ on the time interval $[0, T]$ independent of λ, and that if the initial datum is incompressible datum, then the solutions converge as $\lambda\to\infty$ uniformly on $[0, T]$ to a solution of the incompressible Euler equation.

The aim of the present paper is to show that even if initial datum is not incompressible, the limit still exists and satisfies the incompressible Euler equation. However, the uniform convergence breaks near $t=0$, due to the development of initial layer.

To state our result more precisely, we put, as in [5],

$$p(t, x)=\bar{p}+\lambda^{-1}q(t, x), \quad p_0(x)=\bar{p}+\lambda^{-1}q_0(x),$$

where $\bar{p}$ is an arbitrarily fixed positive number. Set $u=(q, v)$ and $u_0=(q_0, v_0)$. Let H^s denote the Sobolev space $H^s(\boldsymbol{R}^n)$ with norm $\|\cdot\|_s$. Throughout the paper, we take $s\geqq s_0+1$, $s_0=[n/2]+1$. The following theorem is the part of results from [5] which is relevant to us.

Theorem 1.1 ([5]). (i) *For any C_0, $k_0>0$, there exist two positive numbers*

Communicated by Prof. Yamaguti Dec. 27, 1984.