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On Whittaker vectors for generalized Gelfand-Graev representations of semisimple Lie groups

By

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§ 0. Introduction

0.1. Historical background. Early in the 1960's, I. M. Gelfand and M. I. Graev [3] attempted to construct and classify irreducible representations of Chevalley groups over a finite field through irreducible decompositions of the representations induced from characters of a maximal unipotent subgroup. Such an induced representation is called a Gelfand-Graev representation if the character is non-degenerate. They showed that this representation is multiplicity free.

The Gelfand-Graev representations for real semisimple Lie groups are defined in the same way. J. A. Shalika [12] extended the above multiplicity one theorem to quasi-split linear semisimple Lie groups (more generally, to such groups over a local field). For Chevalley groups, H. Jacquet [6] constructed intertwining operators from the principal series representations to the Gelfand-Graev representations through analytic continuation of an integral operator, so called Whittaker integral. G. Schiffmann [13] treated the problem of analytic continuation of Whittaker integral for linear semisimple Lie groups of real rank one. Using his results, M. Hashizume [4] dealt with it for reductive algebraic groups over $\boldsymbol{R}$ of higher rank for the spherical principal series representations.

Recently, N. Kawanaka [8] introduced, generalizing the idea of Gelfand-Graev, the generalized Gelfand-Graev representations of Chevalley groups over a finite field, and proved Ennola duality using their characters. As was suggested in [8], these representations seem to give us more precise informations on irreducible representations than those given only by Gelfand-Graev representations.

0.2. As a first step of our study of the generalized Gelfand-Graev representations of real semisimple Lie groups, we extend in this article the above results for Gelfand-Graev representations to the generalized ones. Standing at the same point of view as [4], we treat intertwining operators from the principal series representations to the contragredient representations of generalized Gelfand-Graev representations.

This article consists of two parts. In the first part §§ 2~3, we deal with a uniqueness property of intertwining operators. The main result of Part I is

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