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A characterization of the finely harmonic morphism in R^n

Dedicated to Professor Yukio Kusunoki on his 60th birthday

By

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Introduction.

B. Fuglede [10] gave a characterization of the harmonic morphism in $\boldsymbol{R}^n$ as follows:

Theorem A (Fuglede). *For a continuous mapping φ from a domain U ($\subset \boldsymbol{R}^n$, $n \geqq 2$) into $\boldsymbol{R}^m$ ($m \geqq 2$), the followings are equivalent:*

(i) *φ is a harmonic morphism on U.*

(ii) *The components φ_j of φ ($1 \leqq j \leqq m$), $\varphi_i \varphi_j$ ($i \neq j$) and $\varphi_i^2 - \varphi_j^2$ are harmonic in U.*

(iii) *The components φ_j of φ ($1 \leqq j \leqq m$) are harmonic in U, and $\nabla \varphi_i \cdot \nabla \varphi_j = \delta_{ij} |\nabla \varphi_1|^2$ on U.*

Recently Fuglede introduced the notion of finely harmonic functions in the potential theory on harmonic spaces and he studied finely harmonic morphisms (cf. [7], [8] and [9]).

In this paper we treat a problem of the same type as Theorem A for finely harmonic morphisms in $\boldsymbol{R}^n$. And we obtain the following theorem which is an extension of Theorem A.

Theorem 1. *For a finely continuous mapping φ from a finely open set U ($\subset \boldsymbol{R}^n$, $n \geqq 2$) into $\boldsymbol{R}^m$ ($m \geqq 2$), the followings are equivalent:*

(i) *φ is a finely harmonic morphism on U.*

(ii) *The components φ_j of φ ($1 \leqq j \leqq m$), $\varphi_i \varphi_j$ ($i \neq j$) and $\varphi_i^2 - \varphi_j^2$ are finely harmonic in U.*

(iii) *The components φ_j of φ ($1 \leqq j \leqq m$) are finely harmonic in U, and*

$$\nabla \varphi_i \cdot \nabla \varphi_j = \delta_{ij} |\nabla \varphi_1|^2 \quad \text{a.e. on } U,$$

where $\nabla \varphi_i$ is the gradient defined in Proposition 1 of §1.

This theorem will be proved by a probabilistic method. For that purpose we give a probabilistic characterization of the finely harmonic morphism in $\boldsymbol{R}^n$.

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