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On Hadamard gap series and errata to Plessner, Julia, and ρ^*-points

Dedicated to Professor Yukio Kusunoki on his 60th birthday

By

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1. Introduction.

This note contains three purposes: First, we present an alternative solution to our previous problem on gap series raised in this journal (Kyoto University 1978). Second, we give a short proof of Fuchs Theorem on gap series. Finally, we correct three of our theorems which were pointed out by Gavrilov (from Moscow State University) to whom the author is indebted.

Let $D=\{z:|z|<1\}$ be the unit disk, $C=\{z:|z|=1\}$ the unit circle, and $f(z)$ a function meromorphic in D. As in K. Noshiro [19, p. 87], we say that $f(z)$ is normal in D if and only if

$$(1-|z|^2)|f'(z)|/(1+|f(z)|^2)\leqq M, \qquad \text{for all} \quad z\in D,$$

where M is a constant independent of points z in D.

In this journal [11, Theorem 7], we proved the following necessary and sufficient conditions of gap series to be normal in D.

Theorem 1. *Let $f_m(z)=\sum_{k=0}^{\infty} n_k^m z^{n_k}$, where $n_{k+1}/n_k\to\infty$, as $k\to\infty$, then f_m is normal if $m\leqq 0$ and f_m is not normal if $m\geqq 1$.*

We conjectured [11, p. 188] that f_m is normal if and only if $m\leqq 0$, where $n_{k+1}/n_k\to q>1$. We posed this problem for $q=\infty$ in Detroit Meeting and it was solved by L. R. Sons [23]. The general case $q>1$ was finally solved by the following theorem of Sons and Campbell [24].

Theorem 2. *Let $f(z)$ be a Hadamard gap series defined by*

$$(1) \qquad f(z)=\sum_{k=0}^{\infty} c_k z^{n_k}, \quad n_{k+1}/n_k\geqq q>1,$$

where the series is convergent in D, then f is normal if and only if $\limsup|c_k|<\infty$.

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