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Existence, uniqueness and analyticity of solutions to boundary value problems for equations of mixed type in a half space

Dedicated to Professor SIGERU MIZOHATA on his sixtieth birthday

By

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§1. Introduction and statements of results.

We consider the following boundary value problem

$$(P)\qquad \begin{cases} \dfrac{\partial^2 u}{\partial x^2}+q(x)\dfrac{\partial^2 u}{\partial t^2}=0\,, & (x,\, t)\in(0,\, \infty)\times R\,, \\ \lim\limits_{x\to 0} u(x,\, t)=g(t) \quad\text{and}\quad \lim\limits_{x\to\infty} u(x,\, t)=0\,, & t\in R\,. \end{cases}$$

The coefficient $q(x)$ is assumed to be a real-valued bounded smooth function on $[0, \infty)$ throughout this paper. We assume

$$(C)\qquad \overline{\lim_{x\to\infty}}\, q(x)<0\,.$$

In our previous work [4] we showed some existence theorems for the above problem (P) with $\dfrac{\partial^2}{\partial^2 x}$ replaced by $\varDelta$, the Laplacian in a higher dimensional space, where $q(x)$ satisfies (C) or $\underline{\lim}_{x\to\infty}\, q(x)>0$. Here confining ourselves to the condition (C) we gain an insight into the problem (P) to obtain the existence and uniqueness theorem of the solution in a fairly distinct and self-contained way and exhibit the analyticity in t of the solution $u(x, t)$ for any fixed x larger than $\inf\{x;\, q(x)>0\}$. We mention also to the existence of solutions satisfying zero boundary data.

Notation. $g(t)$ is said to belong to H^k_γ if $e^{-\gamma t}g(t)\in H^k$, $(k>-\infty)$. We note $\|g\|^2_{H^k_\gamma}\equiv\|g\|^2_{\gamma,k}=\sum_{j=0}^{k}\int_{-\infty}^{\infty}\left|\dfrac{d^j}{dt^j}(e^{-\gamma t}g(t))\right|^2 dt$ for integer $k\geq 0$ and $H_\gamma=H^\infty_\gamma=\bigcap_{k=0}^{\infty}H^k_\gamma$. Denote $g(t)\in B^k_\gamma$ if $e^{-\gamma t}g(t)\in B^k$, and $|g|_{B^k_\gamma}=|g|_{\gamma,k}=\sum_{j=0}^{k}\sup_{t\in R}\left|\dfrac{d^j}{dt^j}(e^{-\gamma t}g(t))\right|$ and $B_\gamma=\bigcap_{k=0}^{\infty}B^k_\gamma$. $f(x, t)\in C^k([0, \infty); H)$ means that $f(x, t)$ is k-times continuously differentiable on $[0, \infty)$ and $\lim_{x\to\infty} f(x, t)=0$ with values in H.

Theorem 1. *Suppose that $q(x)$ satisfies (C). Then there exists a set*

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