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On sojourn times, excursions and spectral measures connected with quasidiffusions

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1. Introduction.

Let $X=(X_t)_{t\geqq 0}$ be a quasidiffusion on the real line with natural scale and speed measure m, i.e. a strong Markov process the infinitesimal generator of which is a restriction of the generalized second order differential operator $D_m D_x$.
Examples are diffusions and birth-and death-processes. We shall consider excursions of X from 0 over some level $a>0$ and study the sojourn time T which X spends over $a'(a'\in[0, a])$ during such an excursion.
If $a'=a$, then T has a mixed exponential distribution. The mixing measure can be identified with the normalized spectral-measure of a so-called dual string (see Proposition 3.2).
We consider the process $(P_t(a, a'))_{t\geqq 0}$where $P_t(a, a')$ is equal to the time which X spends over a' during excursions of X from 0 to a occuring before the local time $l(\cdot, 0)$ of X at zero equals t. If $a'>0$ it turns out to be a randomly stopped compounded Poisson process (Theorem 4.4).
If $a'=0$ or $a'=a$ its Laplace transform can be calculated by the spectral measures of some corresponding dual strings (see point 4.2 below). In paritucIar, if ν denotes the Ito excursion law of X at zero, V the "peak" of an excursion, we obtain the Laplace transform of the measure $\nu(T\in dt, V\geqq a)$ (Corollary 4.6). Finally a connection between the trajectories of X and some spectral measures via the local time of X is derived (Proposition 4.10). This extends a result of Ito, McKean [5] for diffusions.

2. Quasidiffusions, strings and spectral measures.

In this chapter we summarize some definitions and facts on quasidiffusions and related topics which are necessary in the sequel. Proofs are omitted, details can be found e.g. in [2, 3, 5, 6, 9-13] or can be easily derived from those.
We denote by R the real line, by $\mathfrak{B}$ the σ-algebra of its Borelian subsets and by K the set of complex numbers. Let $b\mathfrak{B}$ be the set of all bounded measurable real functions on R.

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