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Differential closure of differential field of positive characteristic

By

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0. Introduction.

Let I be a set of indices and K a differential field of positive characteristic p with aset of (commutative and iterative higher) derivation oparators $\Delta=\{\delta_i ; i\in I\}$. We denote an algebraic closure of K by K_a. Every derivation $\delta_i=(\delta_{i\nu} ; \nu\in N)$ $(i\in I)$, N being the set of all natural numbers including zero, has a unique extension derivation to the separably algebraic closure K_s of K in K_a which we denote also by δ_i; moreover, since these extension derivations δ_i $(i\in I)$ are commutative, K_s is uniquely regarded as a differential extension of K (see [1]). By the paper [2] of myself, we get easily the following two theorems about the extensions of the derivations.

Let x be an element of K_a and δ_i any element of Δ. We say that δ_i can be *extended to* x, if δ_i has an extension derivation to some extension of K_s that contains x. For convenience, we shall denote the e-th power of the characteristic p by $p(e)$.

Theorem A. *An element δ_i of Δ can be extended to x if and only if the condition*

$$(1)\qquad \delta_{i\nu}(x^{p(e)})=0 \qquad (0<\nu<p(e))$$

is satisfied for some element $e\in N$ with $x^{p(e)}\in K_s$. When that is so, setting $y=x^{p(e)}$, the subfield

$$K_{s,x}=K_s((\delta_{i,\nu p(e)}y)^{p(-e)} ; \nu\in N)$$

of K_a has a unique extension derivation $\delta'_i=(\delta'_{i\nu} ; \nu\in N)$ of δ_i which is defined by the fermula

$$(2)\qquad \delta'_{i\nu}z=(\delta_{i,\nu p(e)}(z^{p(e)}))^{p(-e)} \qquad (\nu\in N,\ z\in K_{s,x});$$

the equality $K_{s,x}=K_s(\delta'_{i\nu}x ; \nu\in N)$ holds true, and $K_{s,x}$ is the smallest extension of K_s containing x that has an extension derivation of δ_i.

Remark. We see by [1] that the condition (1) is equivalent to the condition

$$\delta_{i\nu}(x^{p(e)})=0 \quad (\nu\in N-\{0\} \text{ with } p(e)\nmid \nu).$$

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