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Theory of Y-extremal and minimal hypersurfaces in a Finsler space

On Wegener's and Barthel's theories

By

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The theory of hypersurfaces in a Finsler space has been first considered by E. Cartan [2] from two points of view. One is to regard a hypersurface as the whole of tangent line-elements and then it is also a Finsler space [9]. The other is to regard it as the whole of normal line-elements and then it is a Riemannian space. J. M. Wegener ([11], [12]) has treated hypersurfaces from the latter viewpoint and dealt in particular with minimal hypersurfaces. E. T. Davies [3] has considered subspaces from the former viewpoint mainly, but referred a little to minimal subspaces. Both of them have pointed out a weak point of their theories that the minimal subspaces are characterized only by the vanishing of the mean curvature provided Cartan's torsion vector vanishes. To overcome this weak point, W. Barthel [1] has proposed a new Finsler connection with surviving torsion tensor (Postulate 5) and obtained a satisfactory result for the time being. B. Su [10] has further developed the theory of minimal subspaces based on Barthel's standpoint.

There is, however, a strange circumstances; Barthel's characteristic equation of minimal hypersurface does not coincide with Wegener's even if Cartan's connection is treated. Moreover the present author is dissatisfied with Barthel's Postulate 6 "The connection is uniquely determined." from the standpoint of the theory of Finsler connections which has been recently developed.

The purpose of the present paper is to give Wegener's and Barthel's theories respective precise formulations. It is indicated here that the symmetry property of Finsler metric really concerns Barthel's theory. It is the most noteworthy result from the viewpoint of recent theory of Finsler connections that every formulation gives rise to the most suitable connection which is different from the well-known connections; in § 4 is the Cartan Y-connection defined from the fundamental function and a non-zero vector field, and the Cartan C-connection is determined in § 8 from the fundamental function alone.

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