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Decay rates of scattering states for Schrödinger operators

Dedicated to Professor Sigeru Mizohata on his 60 th birthday

By

Hiroshi ISOZAKI

Introduction.

In this paper, we derive a best possible decay rate of scattering states for the Schrödinger operator $H=-\Delta+V(x)$ in $L^2(\boldsymbol{R}^n)$ $(n\geqq 2)$ with a long-range potential. We impose the following assumption on $V(x)$:

$$\text{(A)}\quad \begin{cases} V(x) \text{ is a real-valued } C^\infty\text{-function on } \boldsymbol{R}^n \text{ and for some constant } \varepsilon_0>0 \\ D_x^\alpha V(x)=O(|x|^{-|\alpha|-\varepsilon_0}) \quad \text{as} \quad |x|\to\infty \\ \text{for all multi-index } \alpha. \end{cases}$$

Here for $\alpha=(\alpha_1, \cdots, \alpha_n)$, $D_x^\alpha=(\partial/\partial x_1)^{\alpha_1}\cdots(\partial/\partial x_n)^{\alpha_n}$ and $|\alpha|=\alpha_1+\cdots+\alpha_n$.

As is well-known, if f belongs to the absolutely continuous subspace for H, the local position probability of e^{-itH} decays in the sense that for any $R>0$

$$\int_{|x|<R}|e^{-itH}f(x)|^2dx \longrightarrow 0 \qquad \text{as} \quad |x|\to\infty.$$

It is a rather difficult problem to obtain the rate of decay. In order to study it, one usually considers the operator norm of e^{-itH} in various function spaces different from $L^2=L^2(\boldsymbol{R}^n)$. A convenient choice is the so-called weighted L^2-spaces, and one studies the operator norm in L^2 of $\langle x\rangle^{-\sigma}e^{-itH}\langle x\rangle^{-\rho}$ $(\sigma, \rho>0)$, where $\langle x\rangle=(1+|x|^2)^{1/2}$. In our previous work [2], we have already proved some decay rates for e^{-itH}. Combining the result of [2] with the estimates for the parametrix of e^{-itH} introduced in [5] enables us to prove the following

THEOREM 1. *Let $\chi(\lambda)\in C^\infty(\boldsymbol{R}^1)$ be such that for some $d>0$, $\chi(\lambda)=1$ if $\lambda>2d$, $\chi(\lambda)=0$ if $\lambda<d$. Then for any $s\geqq 0$, there exists a constant $C_s>0$ such that*

$$\|\langle x\rangle^{-s}e^{-itH}\chi(H)\langle x\rangle^{-s}\|\leqq C_s(1+|t|)^{-s},$$

for any $t\in\boldsymbol{R}^1$, where $\|\cdot\|$ is the operator norm in L^2.

This estimate is seen to be best possible if one examines the case of $H_0=-\Delta$. One can also allow some local singularities for V. Suppose V is split into two

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