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Canonical duality for unconditioned strong *d*-sequences

By

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§ I. Introduction.

Let A be a commutative ring with $1\neq 0$ and E an A-module. In this paper we present a theorem about an unconditioned strong d-sequence (abbrev. u.s. d-sequence) on E to which we refer as the *canonical duality theory for u.s. d-sequence.*

Our main theorem is:

(1.1) Theorem. *Let A be a commutative ring with $1\neq 0$ and E an A-module. Assume that a sequence $\boldsymbol{a}=a_1, \cdots, a_s$ of elements in A forms a u.s. d-sequence on E. Then for any injective A-module I, the sequence forms a u.s. d-sequence on*

$$\mathrm{Hom}_A(H_{\boldsymbol{a}}^s(E), I),$$

where $H_{\boldsymbol{a}}^s(E)$ stands for the limit of the direct system of Koszul (co-) homology modules

$$H^i(a_1^n, \cdots, a_s^n; E)$$

and mappings

$$\phi^{n,n+1}: H^i(\boldsymbol{a}^n; E) \longrightarrow H^i(\boldsymbol{a}^{n+1}; E),$$

where $\boldsymbol{a}^m$ denotes the system of elements $a_1^m, \cdots, a_s^m$, for an integer $m>0$.

Here we define the (u.s.) d-sequence as;

(1.2) Definition (cf. [Hu]). Let A and E be as in the theorem above. A sequence of elements $\boldsymbol{a}=a_1, \cdots, a_s$ in A is called a *d-sequence* on E if for each $i=1, \cdots, s$ and for any j with $i\leqq j\leqq s$ the following holds,

$$[(a_1, \cdots, a_{i-1})E : a_i a_j]=[(a_1, \cdots a_{i-1})E : a_j].$$

A sequence $\boldsymbol{a}$ is called a *strong* d-sequence on E, if for any integers

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