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On certain d-sequence on Rees algebra

By

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Introduction.

In this paper we study the d-sequences on the associated graded ring of an ideal in a Noetherian local ring.

Let A be a Noetherian local ring with maximal ideal $\underline{m}$ and $\underline{q}=(a_1,\dots, a_r)$ an ideal of A. We define the Rees algebra of $\underline{q}$ as the subalgebra

$$A[a_1X,\dots, a_rX]$$

of the polynomial ring $A[X]$ in the indeterminate X over the local ring A, and denote it by

$$R=R(\underline{q}).$$

A sequence of elements $x_1,\dots, x_r$ of a commutative ring R is called a d-sequence if for all $i\geqq 0$ and $k\geqq i+1$, we have the equality

$$[(x_1,\dots, x_i)\colon x_{i+1}x_k]=[(x_1,\dots, x_i)\colon x_k].$$

Any regular sequence is obviously a d-sequence and every system of parameters in a Buchsbaum ring is a d-sequence ([7] Prop. 1.7.).

Let us state here some remarkable properties of the Rees algebra of an ideal $\underline{q}$ generated by a d-sequence $a_1,\dots, a_r$ of a local ring A.

Firstly, if $a_1,\dots, a_r$ form a regular sequence, then so do a_1, $a_2-a_1X,\dots, a_r-a_{r-1}X$, a_rX in the Rees algebra $R(\underline{q})$. Hence if A is a Cohen-Macaulay ring so is $R(\underline{q})$ ([1]). However, the converse of the above is not true in general. It has been quite an important problem to describe the condition of the Rees ring to be a Cohen-Macaulay ring. This has been partially settled in some papers [2], [5], [8], [10].

Secondary, if $a_1,\dots, a_r$ form a d-sequence, the Rees algebra is isomorphic to the symmetric algebra [7]. By virtue of this fact, J. Herzog, A. Simis and W. V. Vasconcelos have given a homological characterization of a d-sequence [4]. Recently, S. Goto and K. Yamagishi have shown results of a d-sequence in more detail than the above ([3]).

We treat in this paper the following question: If $a_1,\dots, a_r$ form a d-sequence, then do a_1, $a_2-a_1X,\dots, a_r-a_{r-1}X$, a_rX form a d-sequence in the Rees algebra? This is not true in general (see example (4.2)). We give in this paper a sufficient

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