

Logarithmic transformations on elliptic fiber spaces

By

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Introduction.

In [6], Kodaira introduced the notion of *logarithmic transformations* and showed that any elliptic surface possessing multiple singular fibers can be reduced to an elliptic surface free from multiple fibers by means of *logarithmic transformations*. In this paper, we will generalize this logarithmic transformations on an elliptic threefold.

The difficulty is that we cannot perform logarithmic transformations along arbitrary divisors on the base space. So the following problem is fundamental.

Problem. Given an elliptic threefold $f: X \longrightarrow S$ over S and a divisor C on S , define “logarithmic transformations along C ” and give necessary and sufficient conditions to perform logarithmic transformations along C .

Such an attempt was first done by Perrson [8] and later developed by Nishiguchi [7], Ueno [9] and the present author. They used logarithmic transformations to construct strange non-Kähler degeneration of surfaces. In [3], the author found the simpler method to construct them.

In §1, we shall review the theory of logarithmic transformations on an elliptic surface. In §2, we shall define logarithmic transformations along divisors and give partial answers to the above problem. In §3, as an application of theorem (2.1), we shall construct *examples of non-Kähler degenerations of elliptic surfaces*.

In §4, we shall consider logarithmic transformations in the case where the divisors have only normal crossings. And we shall construct an elliptic threefold which has a multiple fiber of type mI_0 and a singular fiber of type I_0^* along the divisor which are crossing normally.

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Notation and convention.

By an elliptic fiber space $f: V \longrightarrow W$, we mean that f is a proper surjective