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Some remarks on the positivity of fundamental solutions for certain parabolic equations with constant coefficients

By

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§1. Introduction and results.

It is well known that the fundamental solution $(4\pi t)^{-n/2}\exp(-|x|^2/4t)$ of the heat operator $\frac{\partial}{\partial t}-\triangle$ is nonnegative. In this note we will show that this property does never hold for the parabolic equations of higher order with respect to the space variables. Here we will restrict ourselves to the case of single equations with constant coefficients. The general case of parabolic systems with variable coefficients will be treated in the other paper by the latter author.

Let us consider the Cauchy problem

$$\begin{cases} (1.1) & Lu\equiv\frac{\partial}{\partial t}u(t,x)-\sum_{|\alpha|\leqq 2m}a_\alpha\left(\frac{\partial}{\partial x}\right)^\alpha u(t,x)=0 \qquad 0<t\leqq T(<\infty),\ x\in \boldsymbol{R}^n, \\ (1.2) & u(0,x)=u_0(x). \end{cases}$$

We assume that:

(i) *m is a positive integer and a_α are real constants,*

(ii) *L is parabolic, i.e. there exists a positive constant δ such that for any $\xi\in\mathbf{R}^n$ we have*

$$A_{2m}(i\xi)\equiv\sum_{|\alpha|=2m}a_\alpha(i\xi)^\alpha\leqq-\delta|\xi|^{2m}.$$

We say that $E(t,x)$ is a fundamental solution of L if it satisfies

$$LE(t,x)=0 \qquad 0<t\leqq T,\ x\in\mathbf{R}^n,$$

and $\lim_{t\to+0}E(t,x)=\delta(x)$,

where $\delta(x)$ is Dirac's delta. Since the coefficients of L are constant, one of the fundamental solutions is explicitly given by means of the Fourier transform:

$$E_0(t,x)=\int\exp\{ix\xi+t\{A_{2m}(i\xi)+A'(i\xi)\}\}d\xi,$$

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