Classical solutions for a class of degenerate elliptic operators with a parameter

By

Reiko SAKAMOTO

Boundary value problems are generally investigated for elliptic differential operators, degenerating on the boundary of a domain ([1], [2], [3], etc.), which are well posed in appropriate Hilbert spaces. On the other hand, classical solutions seem little investigated except for equations of second order ([4], [5], etc.). In this paper, we consider a class of degenerate elliptic differential operators with a positive parameter, and we seek classical solutions, restricting the parameter small enough.

In §1, the regularity of solutions are considered for F-type operators, analogously in [6]. In §2, two types of half space problems are set for F-type operators corresponding to the location of the invertible zone. In §3, the existence of solutions for half space problems is considered for F-type elliptic operators with a parameter, using the energy estimates for adjoint operators ([7]). In §4, some examples of 4th order operators are given.

§1. Regularity for F-type operators.

1.1. F-type operators.
Let $x=(x_1, x_2,..., x_n)=(x_1, x')\in R^n$, and let

$$A(x;\ D)=\sum_{\nu}a_{\nu}(x)D^{\nu}=\sum_{j=0}^{m}a_j(x;\ D')\ D_1^j,$$

where

$$D=(D_1, D_2,..., D_n)=(D_1, D'),\ D_j=D_{x_j}=-i\frac{\partial}{\partial x_j},$$

$$a_{\nu}(x)\in\mathscr{B}^{\infty}(R^n),\ a_m(x;\ \xi')\not\equiv 0\ \text{near}\ x_1=0,$$

and $a_{j_0}(0, x';\ \xi')\not\equiv 0$ for some $j_0(0\leq j_0\leq m)$. Let l_j be an integer such that

$$D_1^l a_j(0, x';\ \xi')\equiv 0\ \text{for}\ l=0, 1,..., l_j-1,$$

and