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Differential operators on locally compact groups

By

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Introduction

The differential operators on a C^∞-manifold are closely related to the distribution theory on it and can be defined simply by local character (e.g. [7]). F. Bruhat [5] showed that a notion of differential operators can be introduced also on any locally compact group G on the basis of his distribution theory. The differential operators in Bruhat's sense act on his space $\mathcal{D}(G)$ of compactly supported regular functions, and take a quite natural form consistent with those on Lie groups ([5], p. 66). But his very method of defining them seems somewhat intricate and unrefined. In the present paper we intend to get the "differential operators" on G rather by local character. Here, as the base space on which they act, we take the C^∞-class $\mathcal{E}_\infty(G)$ (see below) on G rather than $\mathcal{D}(G)$ or $\mathcal{E}(G)$, the Bruhat space of all regular functions.

The substance of the n-times derivable distributions on G in Bruhat's sense ($n=\infty, 1, 2, \cdots$) ([5], p. 67) has been left unknown in case G is not locally connected. On the other hand, we have the C^n-classes on G, denoted by $\mathcal{E}_n(G)$ ($n=\infty, 1, 2, \cdots$) and defined by using one-parameter subgroups of G, as natural generalizations of those on Lie groups. They were first introduced in J. Riss [12] for the abelian case and lately generalized to any G in H. Boseck, G. Czichowski and K.P. Rudolph [3].(1) $\mathcal{E}(G)$ is included in $\mathcal{E}_\infty(G)$. Let $\mathcal{D}_n(G)$ be the linear subspace of $\mathcal{E}_n(G)$ consisting of the compactly supported functions. We equip each $\mathcal{D}_n(G)$ (resp. $\mathcal{E}_n(G)$) with an inductive limit (resp. projective) topology analogous to that of $\mathcal{D}(G)$ (resp. $\mathcal{E}(G)$). The differential operators on G in our sense are defined as the support-decreasing continuous linear maps on $\mathcal{E}_\infty(G)$ or, what is substantially the same (see section **2**, **1**), on $\mathcal{D}_\infty(G)$. The purpose of the present paper is twofold. One aim is to study the differential operators thus defined in comparison with Bruhat's ones. And the other is to show that for each n, the n-times derivable distributions in Bruhat's sense just coincide with the functions in $\mathcal{E}_n(G)$ no matter G is locally connected or not.

The paper consists of three chapters. Chapter 0 arranges some classical facts concerning the general Lie theory on locally compact groups ([9]) and their dimension. The contents of Chapters 1 and 2 are as follows.

The arguments on $\mathcal{D}_n(G)$ and $\mathcal{E}_n(G)$ in [3] are not necessarily enough for

(1) In the notation of [3], $\mathcal{E}_n(G)$ is designated as $C^n_w(G; C)$ ($n=\infty, 1, 2, \cdots$).

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