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Finite multiplicity theorems for induced representations of semisimpmle Lie groups II

—Applications to generalized Gelfand-Graev representations—

By

Hiroshi YAMASHITA

Introduction

This paper is the sccond part of our work on finite multiplicity property for induced representations. We give in this article finite multiplicity theorems for generalized Gelfand-Graev representations of semisimple Lie groups, applying the results of the first part [32] (referred as [I] later on).

Let G be a connected semisimple Lie group with finite center. In [I], we generalized the result of van den Ban [1] on finiteness of multiplicities in the Plancherel formula associated to a semisimple symmetric space, developing the theory of spherical functions in a much more general setting. Furthermore, we gave there nice sufficient conditions for an induced representation of G to have finite multiplicity property. These criterions enable us to understand, in a unified manner, many finite multiplicity theorems for induced representations, obtained in different situations ([2], [8], [26], etc.).

Among others, we are interested in the following important example. Let $G=KA_pN_m$ be an Iwasawa decomposition of G. Then N_m is a maximal unipotent subgroup of G. We showed in [I] that the induced representation $\mathrm{Ind}_{N_m}^G(\xi)$ (differentiably ($=C^\infty$-) or unitarily ($=L^2$-) induced) has finite multiplicity property for any one-dimensional representation (=character) ξ of N_m (see [I, 4.2]). As wes suggested in Appendix of [I], the study of such an induced representation is reduced, to a large extent, to that of $\mathrm{Ind}_{N_m}^G(\xi)$ with a non-degenerate character ξ. The latter representation is called a *Gelfand-Graev representation* (=GGR for short). According to Shalika [26], the GGRs have a remarkable property: the unitarily induced GGRs are of multiplicity one if G is linear and quasi-split (cf. [I, Theorem 4.5]). These GGRs have been playing an important role not only in the representation theory itself but also in the theory of automorphic forms. (For the historical background of the study of GGRs, we refer to [31, 0.1].)

But, in the representation theoretical point of view, GGRs are not large enough to understand all the irreducible representations through them. In other words, there exist numbers of irreducible representations of G that never "occur"

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