J. Math. Kyoto Univ. (JMKYAZ)
28-4 (1988) 695-749

Representations of Lie superalgebras I

Extensions of representations of the even part

By

Hirotoshi FURUTSU and Takeshi HIRAI

Introduction.

Lie superalgebras are becoming important both in mathematics and in physics. The classification of finite-dimensional simple Lie superalgebras was done by Kac in [8] and also by Kaplansky in [11]. Kac also studied the finite-dimensional representations, especially character formulas for them, in [9] and [10]. The infinite-dimensional representations are much more interesting as in the case of usual Lie groups. Unitary (or unitarizable) representations are of particular interest and importance, dominantly in physical applications. As is well known, the classification and the construction of irreducible unitary representations of Lie groups are of great importance in the theory of infinite-dimensional representations. Therefore we intend to study similar problems for (infinite-dimensional) representations of Lie superalgebras from a general point of view.

In this paper we give a definition of unitarity of such representations, which is methematically natural. Then we give a method of constructing irreducible representations of Lie superalgebras. This method gives a standard approach to classifying irreducible (unitary) representations for any Lie superalgebras. In the second half of this paper, we take some simple Lie superalgebras and give the classification and the construction of their irreducible (unitary) representations.

Let $\mathfrak{g}=\mathfrak{g}_0+\mathfrak{g}_1$ be a Lie superalgebra and (π, V) be its representation on a $\boldsymbol{Z}_2$-graded complex vector space $V=V_0+V_1$ in the sense of Kac [8]. Then, on the even part V_0 and also on the odd part V_1 of V, we have representations of a usual Lie algebra $\mathfrak{g}_0$. We consider the converse, expecting to utilize rich results on representations of $\mathfrak{g}_0$. More exactly, we take a representation (ρ, V_0) of $\mathfrak{g}_0$, and then try to construct a representation (π, V) of $\mathfrak{g}$ such that its even part is isomorphic to V_0 as $\mathfrak{g}_0$-modules. We call this (π, V) an extension of (ρ, V_0). We raise some problems concerning this extension.

Problem 1 (Extensions of irreducible representations of $\mathfrak{g}_0$). Take an irreducible representation ρ of $\mathfrak{g}_0$ on a complex vector space V_0. Then, do there exist any irreducible representations (π, V) of $\mathfrak{g}=\mathfrak{g}_0+\mathfrak{g}_1$ extending (ρ, V_0)? If they

Received, June 8, 1987