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Hypoellipticity for infinitely degenerate elliptic and parabolic operators II, operators of higher order

By

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§1. Introduction and results

The present article is a continuation of the previous work [3] concerning the hypoellipticity of second order operators. Here we are mainly concerned with hypoellipticity of differential operators of the form

$$L = D_t^{2m} + f(t) D_x^{2m} + g(t) D_y^{2m} . \tag{1.1}$$

Throughout this paper, we assume that $f(t)$ and $g(t)$ are functions of class C^∞ satisying the following condition:

(A1) i) *$f(0)=g(0)=0$, $f(t)>0$ and $g(t)>0$ for $t\neq 0$.*
ii) *Both of $f(t)$ and $g(t)$ are monotone increasing for $0<t<\delta$, and monotone decreasing for $-\delta<t<0$.*

Then, the same argument as in the proof of theorem 1 of [3] will give the following (we omit its proof):

Theorem 1. *Let L be a differential operator of the form* (1.1) *with $f(t)$ and $g(t)$ satisfying* (A.1). *Assume moreover:*

(A.2) *there exists a constant α with $0<\alpha<1$ such that*

$$\sqrt[2m]{g(\alpha t)}\,|t \log f(t)| \geqq \varepsilon > 0 \qquad \text{for} \quad 0<t<\delta .$$

Then L is not hypoelliptic.

This result gives a necessary condition of hypoellipticity for operator L. Our main purpose of the present paper is to prove that a certain condition (see (A.4) below) almost complimentary to (A.2) is also sufficient for hypoellipticity of L, under the assumption (see (A.3) below) concerning magnitudes of derivatives of $f(t)$ and $g(t)$.

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