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Hyperbolic operators with symplectic multiple characteristics

By

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1. Introduction

Let U be an open set in $\boldsymbol{R}^d$ with coordinates $x'=(x_1, \cdots, x_d)$. Denote by T^*U the contangent bundle on U and by $(x', \xi')=(x_1, \cdots, x_d, \xi_1, \cdots, \xi_d)$ standard coordinates in T^*U. Let I be an open interval containing the origin and set $\Omega=I\times U$. We denote by $(x, \xi)=(x_0, x', \xi_0, \xi')$ standard coordinates in $T^*\Omega$ and

$$D_j = -i\partial/\partial x_j, j = 0, \cdots, d,\ D = (D_0, D'),\ D' = (D_1, \cdots, D_d)\,.$$

Let

$$P(x, D) = D_0^m + \sum_{j=1}^m A_j(x, D')\, D_0^{m-j} \tag{1.1}$$

be a differential operator in D_0 of order m with coefficients $A_j(x, D')$ which are classical pseudodifferential operators of order j defined near $(\hat{x}, \hat{\xi}')=(0, \hat{x}', \hat{\xi}')\in I\times (T^*U\backslash 0)$. We denote by $p(x, \xi)$ the principal symbol of P and we assume that $p(x, \cdot)$ is hyperbolic with respect to dx_0 near $(\hat{x}, \hat{\xi}')$ that is the zeros ξ_0 of $p(x, \xi_0, \xi_0')$ are all real near $(\hat{x}, \hat{\xi}')$. We shall study the microlocal and local Cauchy problem for $P(x, D)$ with data on $x_0=0$.

Denote by Σ the set of real characteristics of order m of P;

$$\Sigma = \{(x, \xi)\in T^*\Omega\backslash 0;\ p(x, \xi) = dp(x, \xi) = \cdots = d^{m-1} p(x, \xi)=0\}\,. \tag{1.2}$$

We assume that

$$\begin{aligned}&\Sigma \text{ is a } C^\infty \text{ manifold near } \rho = (\hat{x}, \hat{\xi}) = (\hat{x}, \hat{\xi}_0, \hat{\xi}') \text{ with the}\\ &\text{tangent space } T_\rho\Sigma \text{ at } \rho \text{ such that } T_\rho S\supset T_\rho^\sigma\Sigma\cap T_\rho\Sigma\end{aligned} \tag{1.3}$$

where $S=\{x_0=0\}$ is a initial surface and $T_\rho^\sigma\Sigma$ is the σ orthogonal space of $T_\rho\Sigma$. Here σ is a natural 2 form on $T^*\Omega$ given in any standard coordinates (x, ξ) by

$$\sigma = \sum_{j=0}^d d\xi_j\wedge dx_j\,.$$

Note that if $T_\rho\Sigma$ is a symplectic subspace, that is $T_\rho^\sigma\Sigma\cap T_\rho\Sigma=\{0\}$, this condition

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