J. Math. Kyoto Univ. (JMKYAZ)
29-4 (1989) 529-550

On the strongly hyperbolic systems II

—A reduction of hyperbolic matrices—

By

Hideo YAMAHARA

§1. Introduction

This article is the continuation of the previous paper [12]. We shall study the strongly hyperbolic systems ($m\times m$-matrix) in more general cases.

Let $\Omega=(-T, T)\times R_x^l$ and we shall consider the Cauchy problem:

$$(1.1)\qquad \begin{cases} L[u]=\partial_t u-\sum_{k=1}^{l} A_k(t, x)\partial_{x_k}u-B(t, x)u=0 & \text{on } \Omega, \\ u(t_0, x)=u_0(x), \quad -T<t_0<T, \end{cases}$$

where $u(t, x)$ and $u_0(x)$ are m-vectors.

We consider (1.1) in the C^∞-category. Let $L_0=\partial_t-\sum_{k=1}^{l} A_k(t, x)\partial_{x_k}$, then we say that L_0 is a strongly hyperbolic system when the Cauchy problem (1.1) is uniformly C^∞-wellposed for any lower order term $B(t, x)$. For details see [12].

When the coefficients $A_x(t, x)$ are constant or the multiplicites of the characteristic roots of $A(t, x;\xi)=\sum_{k=1}^{l} A_k(t, x)\xi_k$ are constant for any $(t, x;\xi)\in\Omega\times R_\xi^l\setminus\{0\}$, we know the necessary and sufficient conditions for L_0 to be a strongly hyperbolic system ([3], [5]). On the other hand if we do not impose the assumptions on the characteristic roots in the case of variable coefficients, the situation will be much more complicated.

In [12] the author gave a necessary condition without any assumptions of the characteristic roots. But, in it, we assumed that the rank of $(\lambda I-A(t, x;\xi))=m-1$, where $\det(\lambda I-A(t, x;\xi))=0$. And the necessary condition for L_0 to be a strongly hyperbolic system was that the multiplicities of the characteristic roots are at most double at every point $(t, x;\xi)$.

It seems that the difficulties specific for systems will be appear when we drop the above assumption of rank. And instead of the above condition, if L_0 is a strongly hyperbolic system then it will hold that the orders (sizes) of the Jordan's blocks for any characteristic roots must be at most two at any point $(t, x;\xi)$. We will prove the above result in some restricted cases. Moreover when the orders of the Jordan's blocks are equal to two at a certain point we can give the following example.

Example. $L_0=\partial_t-A(t)\partial_x \qquad (l=1)$,

Communicated by Prof. S. Mizohata December 21, 1987