J. Math. Kyoto Univ. (YMKYAZ)
30-1 (1990) 133-148

L-valued 1-jet de Rham complexes and their induced spectral sequences

By

Takeshi USA

§ 0. Introduciton

In [U-4], we raised several problems on a system of projective equations of a projective submanifold X. Those problems were heavily concerned with a given section of a given vector bundle E on X and its induced cohomology classes of the vector bundle valued cohomology groups $H^q(X, \Omega_X^p(E))$. Hence, we need a suitable means to study each cohomology class of $H^q(X, \Omega_X^p(E))$.

As is well-known, there are many works on geometric explanations of the cohomology classes of the cohomology groups $H^q(X, \Omega_X^p)$. Nevertheless, in spite of their importance, we do not have much knowledge on the geometric meanings of the cohomology classes of the E-valued cohomology groups $H^q(X, \Omega_X^p(E))$. And this fact has a lot to do with that, though we have Hodge spectral sequence from $H^q(X, \Omega_X^p)$ to $H^{p+q}(X, \mathbb{C})$, we do not have spectral sequences from "E-valued" cohomology groups to some topological cohomology groups. The difficulty for constructiong such a spectral sequence arises from the fact that in general we can not let the following sequence:

$$0 \longrightarrow E \xrightarrow{\nabla} \Omega_X^1 \otimes E \xrightarrow{\nabla} \cdots \cdots \xrightarrow{\nabla} \Omega_X^n \otimes E \longrightarrow 0$$

be a complex by introducing a connection ∇. Roughly speaking, the obstruction in making a complex with a connection ∇ is described by the curvature operator Θ for the connection ∇. This curvature operator Θ determines the Chern classes of E which are invariants of E. Hence, this approach has serious difficulty.

Thus, in view of our original purpose, we intend to construct a double spectral sequence depending on a given section of E by using another method (cf. (4.5) Theorem). The key idea is to use the sheaves of L-valued 1-jet forms on the projective bundle $P(E)$, in making a suitable complex with a differential operator of first order (cf. Added in proof).

We have some applications of our spectral sequences, for which we need complicated calculation, and we shall publish them elsewhere.

Received April 16, 1988