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A stochastic equation based on a Poisson system for a class of measure-valued diffusion processes

By

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§ 1. Introduction

Measure-valued diffusion processes are of a typical class of infinite dimensional diffusion processes, which arise in various fields such as mathematical biology and filtering theory. Above all, measure-valued branching diffusions in population dynamics and Fleming-Viot diffusion models in population genetics have been studied extensively by many authors from points of large time behaviors based on analysis of the distribution at fixed time $t \geqq 0$, (cf. [13], [1], [2], [6], [11], [9], [4]).

In the present paper we are concerned with probabilistic structure of sample paths for a class of measure-valued diffusion processes including measure-valued branching diffusions and Fleming-Viot diffusion models. For this purpose we will formulate a stochastic equation based on a Poisson system associated with excursion laws of one-dimensional continuous state branching diffusions, which gives an intuitive and comprehensible description of a class of measure-valued diffusion processes and makes the sample path structure clearly observed. Furthermore, by solving the stochastic equation we can provide a new interesting class of measure-valued diffusion processes.

Let S be a basic space that is a locally compact separable metric space, $\boldsymbol{B}(S)$ be the Borel field of S, $M(S)$ be the set of bounded measures on S, and $M_1(S)$ be the set of probability measures on S. $M(S)$ and $M_1(S)$ are equipped with the usual weak topology. We denote by $C_b(S)$ and $C_0(S)$ the set of bounded continuous functions on S and the set of continuous functions of S vanishing at infinity, if S is non-compact. In this paper we will discuss diffusion processes on the state spaces $M(S)$ and $M_1(S)$, which we call measure-valued diffusion processes.

Let us consider the following operator L acting on a class of function on $M(S)$:

$$LF(\mu)=\frac{1}{2}\int_S \mu(dx)\frac{\delta^2 F(\mu)}{\delta\mu(x)^2}+\int_S \mu(dx)A\Big(\frac{\delta F}{\delta\mu}\Big)(x) \tag{1.1}$$

where A is a generator of a Markov process on the state space S with the domain $\boldsymbol{D}(A)$, and $\delta F(\mu)/\delta\mu(x)=\lim_{\varepsilon\downarrow 0}(F(\mu+\varepsilon\delta_x)-F(\mu))/\varepsilon$ (if exists). For example if $F(\mu)=f(\langle\mu, \phi\rangle)$, then $\delta F(\mu)/\delta\mu(x)=\phi(x)f'(\langle\mu, \phi\rangle)$.

The domain of L is given by

$$\boldsymbol{D}(L)=\{F(\mu)=f(\langle\mu, \phi_1\rangle, \cdots, \langle\mu, \phi_k\rangle): k\geqq 1,\ \phi_i\in\boldsymbol{D}(A), \text{ and } f\in C_0^2(R^k)\}. \tag{1.2}$$

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