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3-folds with two P^1-bundle structures

By

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In the present paper, the author determines the structure of 3-folds which have two P^1-bundle structures.

Let X be a projective 3-fold defined over an algebraically closed field k. Then, X is said to have two P^1-bundle structures $(S, T; p, q)$ if there are two P^1-bundles $p: X \to S$ and $q: X \to T$ with projective surfaces S, T in the etale topology and moreover if (P) $\dim h(X) = 3$, where h is the morphism: $X \to S \times T$ induced by p and q.

Then we have

Theorem. *Let X be a smooth 3-fold with two P^1-bundle structures $(S, T; p, q)$. Assume that the characteristic of the ground field k is arbitrary. Then, X is one of the followings:*

1) *$S \times_C T$, where S and T are P^1-bundles over a smooth curve C.*
2) *$P(T_{P^2})$, where T_{P^2} is the tangent bundle over P^2.*

The author has already shown the above theorem in the case of characteristic zero in [Sa]. What is important for the proof is to prove that S and T are ruled, which is trivial in characteristic zero. Namely, the essential part is only that a projective surface dominated by a ruled surface is ruled in characteristic zero. (See Remark 1.3.1) But, in the case of positive characteristic, there are many unirational surfaces which are of general type [Za]. Moreover, in the case, there exists even a surface of general type which is regularly dominated by P^2 (See Proposition 2.12 and remark in [E]).

Hence, in order to prove the ruledness of S and T, we prepare two sufficient conditions about the ruledness: Proposition 2.4 and Proposition 2.7 in §2. These propositions leave us the following case: the second Betti number $\beta_2(S, l) = 2$ and K_S is numerically equivalent to zero, if S is not ruled.

Finally, in §3, we can rule out this case thanks to the fact in [Bo + Mu] (See Proposition 3.7 in this paper).

Thus, throughout this paper, the characteristic of the ground field is supposed to be positive.

Notations. We work over an algebraically closed field k of any positive

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