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Erdös-Rényi law for stationary Gaussian sequences

By

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§ 1. Introduction

Erdös and Rényi [11] discovered a new law of large numbers, nowadays called the Erdös-Rényi law. This law states that for an i. i. d. sequence $\{\xi_j;\ j=1, 2, \cdots\}$ with partial sums $S_0=0$ and $S_n=\sum_{j=1}^{n}\xi_j$, if the moment generating function $M(t)=E\exp(t\xi_1)$ exists for all $t\in(0, t_1)$, then for each $\alpha\in\{M'(t)/M(t);\ t\in(0, t_1)\}$ and $c=c(\alpha)$ such that

$$\exp(-1/c)=I(\alpha):=\inf_t M(t)\exp(-t\alpha),$$

we have

$$\lim_{n\to\infty} \boldsymbol{D}(n,\ [c\log n])=\alpha, \quad \text{a. s.,}$$

where

$$\boldsymbol{D}(n,\ k)=\max_{0\leq j\leq n-k}\frac{S_{j+k}-S_j}{k}, \qquad 1\leqq k\leqq n,$$

and $[\cdot]$ denotes the integral part.

Many general versions of the Erdös-Rényi law for i. i. d. sequences have been developed by Book [1]~[2], M. Csörgo [5]~[6], S. Csörgo [7], Deheuvels [8]~[9] and Steinebach [17]~[20] and others.

However, Deo [10] initially developed the original Erdös-Rényi law to a stationary Gaussian sequence under a condition on the correlation function. More precisely, suppose $\{\xi_j;\ j=1, 2, \cdots\}$ is a stationary Gaussian sequence with $E\xi_1=0$, $E\xi_1^2=1$ and $r_n=E\xi_1\xi_{1+n}$, $n=1, 2, \cdots$, such that

$$\lim_{n\to\infty} n^{1+\beta}r_n=0 \qquad \text{for some } \beta>0$$

and

$$0<\sigma^2=1+2\sum_{j=1}^{\infty}r_j.$$

then for each $0<c<\infty$

$$\lim_{n\to\infty} \boldsymbol{D}(n,\ [c\log n])=\sigma\sqrt{2/c}, \text{ a. s..}$$

Our object of this paper is to improve Deo's result and obtain a general form of the Erdös-Rényi law for stationary Gaussian sequences. Our result is as follows: Let $\{\xi_j;\ j=1, 2, \cdots\}$ be a stationary Gaussian sequence with $E\xi_1=0$, $E\xi_1^2=1$ and $r_n=E\xi_1\xi_{1+n}$

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