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Moduli of stable pairs

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Introduction

Let S be a scheme of finite type over a universally Japanese ring Ξ and let $f: X \to S$ be a smooth, projective, geometrically integral morphism. We shall fix an f-very ample invertible sheaf $\mathcal{O}_X(1)$ and a locally free $\mathcal{O}_X$-module E of finite rank. An E-pair is a pair (F, φ) of a coherent sheaf F on a geometric fiber of f and an $\mathcal{O}_X$-homomorphism φ of F to $F \otimes_{\mathcal{O}_X} E$ such that φ induces a canonical structure of $S^*(E^\vee)$-module on F. An E-pair (F, φ) is said to be stable (or, semi-stable) if F is torsion free and if it satisfies the stability (or, semi-stability, resp.) inequality for all φ-invariant subsheaves of F (see § 1). Stable pairs were first introduced by N. J. Hitchin [3] in the case where $S = \mathrm{Spec}(k)$ with k an algebraically closed field and where X is a curve and E is a line bundle. In this case, the moduli spaces of stable E-pairs were constructed by N. Nitsure [10], and W. M. Oxbury studied some properties of the moduli spaces [11]. In higher dimensional cases, C. T. Simpson constructed the moduli spaces of semi-stable E-pairs over an algebraically closed field of characteristic zero [13]. In the method of C. T. Simpson, an E-pair (F, φ) were considered as a sheaf on $Y = \mathrm{Proj}(S^*(E^\vee) \oplus \mathcal{O}_X)$ and the problem was reduced to the study of stable points on $Q = \mathrm{Quot}^H_{\mathcal{O}_Y(-N)^{\oplus m}/Y/S}$ for large integers N, where $\mathcal{O}_Y(1)$ is a very ample invertible sheaf on Y and H is the Hilbert polynomial of F with respect to $\mathcal{O}_Y(1)$. To handle this problem he embedded Q into the Grassmann variety Grass $(H^0(\mathcal{O}_Y(l-N)^{\oplus m}), H(l))$ with l a sufficiently large integer. His proof depends, in essential way, on the boundedness theorem of M. Maruyama (Theorem 4.6 of [8]) which fails to hold in positive characteristic cases. The aim of this article is to construct a moduli scheme of semi-stable E-pairs along the method by D. Gieseker [2], M. Maruyama [6] and [7] and then our results hold good without assuming characteristic zero. The main idea is to find a space which seems as the "Gieseker space" in [2], [6] and [7]. It is the projective space $\mathbf{P}(\mathrm{Hom}_{\mathcal{O}_X}(V \otimes_\Xi (\bigoplus_{i=0}^{r-1} S^i(E^\vee)), L)^\vee)$, where L is a line bundle on X and r is the rank of F. On the other hand, to parametrize E-pairs we have to use a scheme Γ constructed in § 4 instead of Quot-scheme in the case of usual stable sheaves and to study stable points of Γ we have to introduce a morphism of Γ to a projective bundle on $\mathrm{Pic}_{X/S}$ whose fibers are

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