J. Math. Kyoto Univ. (JMKYAZ)
31-1 (1991) 289–306

On the asymptotic behavior of the increments of a Wiener process

By

Norio KÔNO

§ 1. Introduction

For a Wiener process $\{W(t); 0 \leqq t < +\infty\}$, Erdös-Rényi law [5] implies the following strong limit theorem:

$$\lim_{T\to\infty} \sup_{0\leqq t\leqq T-c\log T} |W(t+c\log T)-W(t)|/\log T = \sqrt{2c} \text{ a.s. } (c>0)$$

After them varieties of such limit theorems are proved (for examples, see [3], [4]). Furthermore, P. Révész [8] has investigated much sharper limit theorems using the notion of upper class or lower class.

Definition 1. Let f and g be two real (not necessarily random functions) defined on the positive half line, and we assume that g is a monotone function. Then g is called an upper-upper function of f (briefly, $g \in UUC(f)$) if and only if there exists $t_0 > 0$ such that for all $t > t_0$, $f(t) < g(t)$ holds, and g is called an upper-lower function of f (briefly, $g \in ULC(f)$) if and only if there exists an infinite sequence $t_1 < t_2 < \cdots \to +\infty$ such that $f(t_n) > g(t_n)$ holds for all n.

In this paper, we will give an integral test which determines whether a given determinstic function belongs to UUC or ULC of the folowing functionals of a Wiener process with probability one:

$$X_0(T) = \sup_{0\leqq s\leqq a_T} \sup_{0\leqq t\leqq T-s} |W(s+t)-W(t)|/\sqrt{a_T},$$

$$X_1(T) = \sup_{0\leqq s\leqq a_T} \sup_{0\leqq t\leqq T-a_T} |W(s+t)-W(t)|/\sqrt{a_T},$$

$$X_2(T) = \sup_{0\leqq s\leqq a_T} \sup_{0\leqq t\leqq T-a_T} (W(s+t)-W(t))/\sqrt{a_T},$$

$$X_3(T) = \sup_{0\leqq t\leqq T-a_T} |W(t+a_T)-W(t)|/\sqrt{a_T},$$

$$X_4(T) = \sup_{0\leqq t\leqq T-a_T} (W(t+a_T)-W(t))/\sqrt{a_T},$$

Received July 24, 1989