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Spectral and scattering theory for the Schrödinger operators with penetrable wall potentials

Dedicated to Professor Tosio Kato on his 70th birthday

By

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§ 0. Introduction

In this paper we shall consider the Schrödinger operator with a penetrable wall potential in $\boldsymbol{R}^3$ formally of the form

$$H_{formal} = -\Delta + q(x)\delta(|x| - a),$$

where $q(x)$ is real and smooth on $S_a = \{x; |x| = a\}$ $(a > 0)$ and δ denotes the one-dimensional delta function. This operator is said to provide a simple model for the α-decay (Petzold [15]). Other applications may be found in the references cited in Antoine-Gesztesy-Shabani [3]. Dolph-McLeod-Thoe [5] treated this operator ($q(x) \equiv$ const.) with concern for the analytic continuation of the scattering matrix, yet at the formal level.

The first problem one meets is to define properly H_{formal} as a selfadjoint opertor in $L_2(\boldsymbol{R}^3)$. For this purpose, let us consider the quadratic form h (which is associated with H_{formal})

$$h[u, v] = (H_{formal}\, u, v) = (\nabla u, \nabla v) + (q\gamma u, \gamma v)_a,$$

$$Dom[h] = H^1(\boldsymbol{R}^3).$$

Here γ is the trace operator from $H^1(\boldsymbol{R}^3)$ to $L_2(S_a)$, $Dom[h]$ denotes the form domain of h, $(\ ,\ )$ means the $L_2(\boldsymbol{R}^3)$ inner product, $(\ ,\ )_a$ the $L_2(S_a)$ inner product, and $H^m(G)$ the Sobolev space of order m over G. h is shown to be a lower semibounded closed form, and thus determines a lower semibounded selfadjoint operator H. More precisely, H is seen to be the negative Laplacian with the boundary condition

$$q(x)(\gamma u)(x) - \left\{\frac{\partial u}{\partial n_+}(x) + \frac{\partial u}{\partial n_-}(x)\right\}|_{S_a} = 0,$$

where $n_+(n_-)$ denotes the outward (inward) normal to S_a. We should note here that while h is a "small" perturbation of h_0, which is defined by

$$h_0[u, v] = (\nabla u, \nabla v),\ Dom[h_0] = H^1(\boldsymbol{R}^3),$$

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