J. Math. Kyoto Univ. (JMKYAZ)
31-1 (1991) 191-217

Erdös-Rényi-type laws applied to Gaussian processes

By

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1. Introduction

The Erdös-Rényi law of large numbers applied to standard normal random variables is as follows:

Theorem A. *Let* $\{\xi_j; j = 1, 2, \dots\}$ *be independent standard normal random variables with partial sums* $S_0 = 0$ *and* $S_n = \xi_1 + \cdots + \xi_n$. *Then for each* $c > 0$

$$\lim_{n\to\infty} \max_{0 \le j \le n - [c \log n]} \frac{S_{j+[c \log n]} - S_j}{[c \log n]} = \sqrt{2/c}, \qquad \text{a.s.}$$

where $[y]$ *denotes the greatest integer not exceeding* y.

Many extensions and developments, in various directions, of Erdös-Rényi-type laws for i.i.d.r.v.'s have been obtained in [2], [9], [14], [25–27] and others.

Recently, Choi [5] has extended the Erdös-Rényi law to stationary Gaussian sequences in dependent situations, under milder conditions than those of Deo [10].

The Erdös-Rényi law for Wiener processes has the following form:

Theorem B. *Let* $W(t)(0 \le t < \infty)$ *be a standard Wiener process. Then for each* $c > 0$

$$\lim_{T\to\infty} \sup_{0 \le t \le T - c \log T} \frac{W(t + c \log T) - W(t)}{(2c)^{1/2} \log T} = 1, \qquad \text{a.s.}$$

Csörgő and Révész [7] obtained the following result for the increments of $W(t)$:

Theorem C. *Let* $a_T(0 \le T < \infty)$ *be a nondecreasing function of* T *for which*

(1.1) $$0 < a_T \le T$$

(1.2) $$a_T/T \text{ is nonincreasing}$$

(1.3) $$\lim_{T\to\infty} \frac{\log (T/a_T)}{\log \log T} = \infty.$$

Communicated by Professor S. Watanabe, June 21, 1989