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Partially conformal qc mappings and the universal Teichmüller space

Dedicated to Professor Kôtaro Oikawa on his 60th birthday

By

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Introduction

Let $M(\Delta)$ be the set of all Beltrami coefficients on the unit disk Δ, that is, it is the set of all bounded measurable functions μ defined on Δ with $\|\mu\|_\infty = \operatorname{esssup}_\Delta|\mu(z)| < 1$. We denote by w_μ the unique quasiconformal (qc) self-mapping of Δ satisfying the Beltrami equation $w_{\bar{z}} = \mu w_z$ and leaving ± 1, i fixed. Two elements μ and ν in $M(\Delta)$ are called equivalent if $w_\mu = w_\nu$ on $\partial\Delta$. The universal Teichmüller space T is defined as the quotient space of $M(\Delta)$ with respect to this equivalence relation. This space T carries a natural metric, called the Teichmüller metric (cf. Lehto [3]), with respect to which the canonical projection $\Phi\colon M(\Delta) \to T$ is open as well as continuous.

Let V be a measurable subset of Δ, and set

$$M(V) = \{\mu \in M(\Delta);\ \mu|_{(\Delta - V)} = 0\}.$$

We denote the Banach space of all integrable holomorphic functions on Δ by A, and the characteristic function of a set Y by $\chi(Y)$. Our first result is a necessary condition for V to insure that the points which can be represented by quasiconformal mappings whose Beltrami coefficients are in $M(V)$ contain a non-empty open set in T.

Theorem 1. *Let V be a measurable subset of Δ with positive measure. If the interior of $\Phi(M(V))$ is not empty, then*

$$\inf\{\|\chi(V)\phi\|_1;\ \phi \in A,\ \|\phi\|_1 = 1\} > 0. \tag{1}$$

We denote the hyperbolic disk with center at $\zeta \in \Delta$ and hyperbolic radius ρ by $D(\zeta; \rho)$, and the hyperbolic area of $Y \subset \Delta$ by $\sigma(Y)$.

Definition 1. *A* measurable subset Y of Δ is *uniformly distributed in mean* if

$$\inf\left\{\frac{\sigma(Y \cap D(\zeta; \rho))}{\sigma(D(\zeta; \rho))};\ \zeta \in \Delta\right\} > 0 \qquad \text{for some } \rho > 0.$$

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