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Homology of the Kac-Moody groups II

Dedicated to Professor Shôrô Araki on his 60th birthday

By

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§ 1. Introduction

Let G be a compact, connected, simply connected, simple Lie group and $\mathfrak{g}$ its Lie algebra. Let $X\langle n\rangle$ be the n-connected cover of the space X. Since $\pi_3(G) \cong \boldsymbol{Z}$ is the first non-trivial homotopy, there is an S^1-fibration

$$S^1 \longrightarrow \Omega G\langle 2\rangle \longrightarrow \Omega G.$$

(Notice that sometimes one likes to write $\Omega G\langle 3\rangle = \Omega(G\langle 3\rangle)$ instead of our $\Omega G\langle 2\rangle = (\Omega G)\langle 2\rangle$.) The homotopy type of the Kac-Moody group $\mathfrak{K}(\mathfrak{g}^{(1)})$ is $\Omega G\langle 2\rangle \times G$. (See [10] and [11].) Since the homology of G is known and $H_*(\Omega G\langle 2\rangle; \boldsymbol{Z})$ is finitely generated, we have only to determine $H_*(\Omega G\langle 2\rangle; \boldsymbol{Z}_{(p)})$ for all prime p to determine $H_*(\mathfrak{K}(\mathfrak{g}^{(1)}); \boldsymbol{Z})$.

The homology of G has non trivial p-torsions if and only if (G, p) is one of the following:

$$(Spin(n), 2)\ n \geq 7,\ (E_6, 2),\ (E_6, 3)$$

$$(E_7, 2),\ (E_7, 3),\ (E_8, 2),\ (E_8, 3),\ (E_8, 5),$$

$$(F_4, 2),\ (F_4, 3) \text{ and } (G_2, 2).$$

In [14], we computed $H_*(\Omega G\langle 2\rangle; \boldsymbol{Z}_{(p)})$ for such (G, p) except $(Spin(n), 2)$ and $(E_6, 2)$.

The purpose of this paper is to determine it for the groups whose homology has no p-torsion. The major problem in the above case is that it is very difficult to compute the Gysin sequence of $\boldsymbol{Z}_{(p)}$-coefficients directly. To avoid this problem, we consider the Bockstein spectral sequence of the Gysin sequence. By using the Serre spectral sequence associated with $\Omega G\langle 2\rangle \to \Omega G \to CP^\infty$, we can prove that the first non trivial p-torsion of $H_*(\Omega G\langle 2\rangle; \boldsymbol{Z}_{(p)})$ is order p for all G. (See Theorem 3.1.) This fact becomes the "seed" of our computation of the above Bockstein spectral sequence and also gives the result for $(E_6, 2)$.

We define $\boldsymbol{Z}_{(p)}$-modules $C(d, p)$ and $L(G, p)$ in § 3. Then the main result is

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