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On a multiplicative structure of BP-cohomology operation algebra

By

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§ 0. Introduction

The BP-theory is obtained as a factor of the p-localized complex cobordism theory, and has a close relation to the theory of p-typical formal group laws. For example, (BP_*, BP_*BP) has a particular algeraic structure, named Hopf-algebroid [10], and we can formulate its left unit, right unit, coproduct and canonical antipodal isomorphism in terms of the formal group law obtained from the complex orientation of the BP-theory. Since the E_2-term of the Adams-Novikov spectral sequence is a cohomology of the Hopf-algebroid (BP_*, BP_*BP), we can obtain many useful information from these formulae.

BP^*BP, which is a dual of BP_*BP, is a cohomology operation algera of the BP-theory, and can be regarded as a kind of (non-commutative) Hopf-algebra. D. Quillen [9] studied its Hopf-algebraic structure, and asserted that $BP^*BP \cong \mathrm{Hom}_{BP_*}(BP_*BP, BP_*) \cong \mathrm{Hom}_{\boldsymbol{Z}}(\boldsymbol{Z}[t_1, t_2, \cdots], BP_*) \cong BP^* \hat{\otimes} R$, where $R = \{r_E\colon E = (e_1, e_2, \cdots)\}$ is a dual basis of $\{t^E = t_1^{e_1} t_2^{e_2} \cdots\}$, the BP_*-free basis of BP_*BP (see also [1]). We call these r_E the Quillen elements. But its multiplicative structure has been expressed as a dual of the comultiplicative structure of BP_*BP, so that the complicatedness of this coproduct formula seems to prevent our intimate studying of the multiplicative structure of BP^*BP. Exceptionally, R. Kane [5], [6] demonstrated some interesting results about BP-operations and Steenrod operations from their behavior under the rationalization and the product formula modulo $(v_1, v_2, \cdots)$.

The purposes of this paper are to describe the complete formula for the product of BP-operations and to study the algebraic structure of BP^*BP by means of this formula.

§ 1. Product formula

BP^*BP is a stable cohomology operation algebra of the BP-theory. This is a dual algebra of $BP_*BP \cong BP_*[t_1, t_2, \cdots]$ $(\deg t_i = 2(p^i - 1))$ because BP_*BP is a free left module over the coefficient ring $BP_* \cong \boldsymbol{Z}_{(p)}[v_1, v_2, \cdots]$ $(\deg v_i = 2(p^i - 1))$.

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