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Stability of foliations of 4-manifolds by Klein bottles

Dedicated to Professor Masahisa Adachi on his 60th birthday

By

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§0. Introduction

This paper is a complement to my paper [8]. Let $\mathrm{Fol}_q(M)$ denote the set of condimension q C^∞-foliations of a closed manifold M. $\mathrm{Fol}_q(M)$ carries a natural weak C^r-topology ($0 \leqq r \leqq \infty$), which is described in [6], [9]. We denote this space by $\mathrm{Fol}_q^r(M)$. We say a foliation F is C^r-stable if there exists a neighborhood V of F in $\mathrm{Fol}_q^r(M)$ such that every foliation in V has a compact leaf. We say F is C^r-unstable if not. We say a foliation in a small neighborhood of F in $\mathrm{Fol}_q^r(M)$ is a small C^r-perturbation of F. It seems to be of interest to determine if F is C^r-stable or not.

In this paper we shall give a sufficient condition for a foliation of a closed 4-manifold by Klein bottles to be C^1-stable. More precisely we have the following.

Theorem. *Let F be a foliation of a closed 4-manifold M by Klein bottles. If $\chi_V(M/F)^2 + \chi(M/F)^2 \neq 0$, then F is C^1-stable.*

1. Foliations of 4-manifolds by Klein bottles

Let M be a closed manifold and F a compact foliation of M of codimension two. By the results of Epstein [4] and Edwards-Millett-Sullivan [3], we have a nice picture of the local behavior of F as follows.

Proposition 1 (Epstein [5]). *There is a generic leaf L_0 with property that there is an open dense saturated subset of M, where all leaves have trivial holonomy and are diffeomorphic to L_0. Given a leaf L, we can describe a neighborhood $U(L)$ of L, together with the foliation on the neighborhood as follows. There is a finite subgroup $G(L)$ of $O(2)$ such that $G(L)$ acts freely on L_0 on the right and $L_0/G(L) \cong L$. Let D^2 be the unit disk. We foliate $L_0 \times D^2$ with leaves of form $L_0 \times \{pt\}$. This foliation is preserved by the diagonal action of $G(L)$, defined by $g(x, y) = (x \cdot g^{-1}, g \cdot y)$ for $g \in G(L)$, $x \in L_0$ and $y \in D^2$, where $G(L)$ acts linearly on D^2. So we have a foliation induced on $U = L_0 \times D^2/G(L)$. The leaf corresponding to $y = 0$ is $L_0/G(L)$. Then there is a C^∞-imbedding $\varphi\colon U \to M$ with $\varphi(U) = U(L)$, which*

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