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Differentially separable extension of positive characteristic p

By

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§ 0. Introduction

It is well known in the field theory that if $a_1, \ldots, a_s$ are separably algeraic elements of an extension of a field K, there exists an element b such that $K(a_1, \ldots, a_s) = K(b)$. It is interesting to see if this result can be extended to differential fields.

In [2], Kolchin showed that for the usual derivation, such an extension can be done under a condition: if elements $a_1, \ldots, a_s$ of a differential extension of a differential field K are differentially separably dependent over K i.e. for each i $(1 \leq i \leq s)$, there is a differential polynomial $F_i(X_1, \ldots, X_s)$ over K such that $F_i(a_1, \ldots, a_s) = 0$ and $(\partial F_i/\partial(\theta_i X))(a_1, \ldots, a_s) \neq 0$ for some differential operator θ_i, there exists an element b such that the differential field $K\langle a_1, \ldots, a_s\rangle$ i.e. the smallest differential extension field of K containing $a_1, \ldots, a_s$ is equal to $K\langle b\rangle$ (see Proposition 9 of Chapter 2 of [2]).

Since Hasse's differentiation gives more natural results than the usual derivation which in some cases gives pathological results for positive characteristic, we show in this paper that by using Hasse's differentiation a formulation of the extension mentioned at the top of this section can also be performed for positive characteristic p.

§ 1. Definitions*)

Let R be a commutative unitary ring containing some field as a unitary subring. A *derivation* δ of R means an iterative higher derivation of infinite rank i.e. an infinite sequence $\delta = (\delta_\nu; \nu \in \mathbf{N}$, the set of natural numbers including 0) of mappings δ_ν of R into R which satisfies the following conditions:

D 1 $\delta_0 = \mathrm{id}_R$ (the identity mapping of R),

D 2 $\delta_\nu(x + y) = \delta_\nu x + \delta_\nu y$,

D 3 $\delta_\nu(xy) = \sum_{\lambda+\mu=\nu} \delta_\lambda x \cdot \delta_\mu y$,

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*) At large see Okugara [4]