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A note on global strong solutions of semilinear wave equations

Dedicated to Prof. Teruo Ikebe on his 60th birthday

By

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Introduction

Let L be a linear hyperbolic partial differential operator of second order with coefficients depending on $t \in [0, \infty)$ and $x = (x_1, \dots, x_n) \in \mathbf{R}^n$. Let Ω be a bounded or unbounded domain in $\mathbf{R}^n$ with a smooth boundary $\partial\Omega$, or the whole $\mathbf{R}^n$. We assume $3 \leqq n \leqq 6$. Let I be an interval in $[0, \infty)$ and let the time variable t run over I. We shall treat the following problem for a semilinear wave equation:

$$\text{[SLP]} \qquad \begin{cases} L[u] + F(u) = 0 & \text{in } I \times \Omega, \\ u(0, x) = f(x), \quad \partial_t u(x, 0) = g(x) & \text{in } \Omega, \\ B u(t, x) = 0 & \text{on } I \times \partial\Omega, \end{cases}$$

where F is a nonlinear term satisfying $|F(u)| \leqq \text{const.}\,(|u| + |u|^{\frac{n}{n-2}})$ as $|u| \to \infty$, and B is a suitable linear boundary operator. If $\Omega = \mathbf{R}^n$, of course we omit the boundary condition and consider the initial value problems. For mixed problems suitable boundary conditions are assigned. The explicit form of the linear hyperbolic operator L and the boundary operator B will not be given, because it is inessential to our argument.

Our aim is to show the existence of a global strong solution $u = u(t, x)$ of [SLP] for $I = [0, \infty)$. For that purpose we also need to solve [SLP] for some sufficiently short time intervals I. In order to solve [SLP] for some I we shall use some established results of the corresponding linear problem [LP],

$$\text{[L P]} \qquad \begin{cases} L[v] = G & \text{in } I \times \partial\Omega, \\ v(0, x) = f(x), \quad \partial_t v(0, x) = g(x) & \text{in } \Omega, \\ B v(t, x) = 0 & \text{on } I \times \partial\Omega, \end{cases}$$

where G is a given inhomogeneous term, and L, B and the data are the same as in [SLP].

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