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Non-existence of positive eigenvalues of the Schrödinger operator in a domain with unbounded boundary

By

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Introduction

In this paper we shall concern ourselves with the solution of the differential equation

$$(0.1)\qquad (-\Delta+q-\lambda)u=0$$

in a domain $D\subset R^n (n\geqq 2)$, where $\Delta=\sum_j(\partial/\partial x_j)^2$, $\lambda>0$, and q is a complex valued function. Define a domain D_α of R^n by

$$(0.2)\qquad D_\alpha=\{x\in R^n \mid x_1>|x|\cos(\alpha\pi/2)\},$$

where $1<\alpha<2$, $|x|=(x_1^2+\cdots+x_n^2)^{1/2}$. We shall prove the following theorem.

Theorem 0.1. *Assume that D is larger than the half space $x_1>0$ in the sense that there exists a constant c with $1<c<2$ such that*

$$(0.3)\qquad D\supset D_c,$$

and assume that q can be written as $q=q_1+q_2$ such that the following conditions (0.4)$\sim$(0.6) *are satisfied.*

(0.4) *q_1 is real valued, of class $C^1(D)$, and*

$$q_1(x)=o(1)\qquad (|x|\to\infty \text{ in } D_c).$$

$$(0.5)\qquad |\nabla q_1(x)|+|q_2(x)|=o(|x|^{-1})\qquad (|x|\to\infty \text{ in } D_c).$$

(0.6) *There exist constants d and $\delta>0$ such that $1<d<c$, and*

$$|\nabla q_1(x)|+|q_2(x)|=O(|x|^{-(2/c)-\delta})\qquad (|x|\to\infty \text{ in } D_c-D_d).$$

In addition assume that q is such that the unique continuation property holds for equation (0.1), *i.e. if a solution u of* (0.1) *vanishes in an open set of D, u vanishes in all of D. Then if a solution u belongs to $L^2(D)$, u vanishes identically: $u\equiv 0$.*

Here it should be noted that in the hypotheses of the theorem we assume no conditions on the values of the solution u on the boundary ∂D of D.

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