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Triple coverings of algebraic surfaces according to the Cardano formula

By

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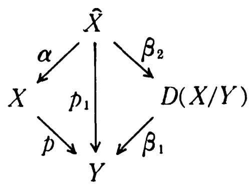
§ 0. Introduction

In this article, we consider a triple covering of an algebraic surface. In case of a cyclic covering, that is, its rational function field is obtained by a cyclic extension of degree 3, its structure is well-known. But in case of a non-Galois covering the structure is not well-known. In [6], R. Miranda obtained some results about a non-Galois triple covering by using a rank 2 vector bundle (called the "Tschirnhausen module"). T. Fujita and R. Lazarsfeld proved a beautiful theorem about a non-Galois triple covering over $\boldsymbol{P}^n$ ($n \geqq 4$) (see [3], [5]). In this paper, we study a non-Galois triple covering by using the Cardano formula. An outline of our method is as follows:

Let $p: X \to Y$ be a finite normal triple covering of a normal variety Y. First, we define the discriminant variety $D(X/Y)$ and the minimal splitting variety $\hat{X}$ associated to the triple covering $p: X \to Y$. For these varieties, we have a commutative diagram:



For details, see § 1 below. To study the triple covering $p: X \to Y$, we study structures of the morphisms $\beta_1: D(X/Y) \to Y$, $\beta_2: \hat{X} \to D(X, Y)$, and $\alpha: \hat{X} \to X$.

Our main results are as follows:

Proposition 3.1. *Let $p: X \to Y$ be a finite totally ramified triple covering of a smooth projective variety Y. Assume that*

(i) *X is smooth,*

(ii) *Y is simply connected.*

Then, p is cyclic, and the branch locus of p is smooth.

Proposition 3.4. *Let $p: S \to \Sigma$ be a finite triple covering where S and Σ are smooth surfaces. Assume that $\Delta(S/\Sigma)$ (the branch locus of p) is an irreducible divisor and has*

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