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Spectra and monads of stable bundles

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§ 1. Introduction

Let $\mathscr{E}$ be a stable vector bundle of rank two on $\mathbf{P}^3$ with $c_1 = 0$ and given c_2. Associated to $\mathscr{E}$ are its spectrum (§ 2) and its minimal monad (§ 3, Proposition 3.2). In this article, we investigate the possible spectra and minimal monads for low values of c_2.

The spectrum χ satisfies three necessary conditions ((S1)–(S3) of § 2). We show that for $c_2 \leq 19$, these conditions are also sufficient for the existence of a stable bundle with that spectrum. The question of existence for larger values of c_2 is left unresolved but it seems reasonable to conjecture, based on the evidence so far, that (S1)–(S3) form necessary and sufficient conditions on a sequence of integers for it to be the spectrum of a rank two stable bundle on $\mathbf{P}^3$ with $c_1 = 0$. When $c_2 \leq 19$, we in fact do more. For each possible spectrum, we produce a bundle $\mathscr{E}$ for which $H^0(\mathscr{E}(1)) \neq 0$. This is also the reason that we stop at $c_2 = 19$. When $c_2 \leq 18$, the bundle $\mathscr{E}$ is constructed from a double structure on a reduced curve. For $c_2 = 19$, there is one spectrum for which $\mathscr{E}$ is constructed via a double structure on a non-reduced curve (in fact, a quadruple structure on a reduced curve). We take this as an indication that the construction gets harder beyond $c_2 = 19$.

We also determine all possible minimal monads when $c_2 \leq 8$, thus completing work begun by Barth [B-1]. We find that some monads listed in his tables do not occur, while others, which had been excluded by his simplifying assumption [B-1], top of p.211, do in fact exist (see § 6). The results are tabulated in § 5.

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§ 2.

Let $\mathscr{E}$ be a stable rank two bundle with $c_1 = 0$ and given c_2 on $\mathbf{P}^3$ (over an algebraically closed field of arbitrary characteristic). Barth and Elencwajg [B-E] have defined (in characteristic zero) the *spectrum* of $\mathscr{E}$, which is a certain sequence of c_2 integers. In [H-1], § 7, a characteristic-free definition of the spectrum is given and the following properties are proved in [H-1] and [H-2]. Let $\chi = \{k_1, k_2, \ldots, k_{c_2}\}$, $k_i \in \mathbf{Z}$ be the spectrum of $\mathscr{E}$. Then χ and $\mathscr{E}$ satisfy:

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