J. Math. Kyoto Univ. (JMKYAZ)
31-3 (1991) 695-711

Local structure of analytic transformations of two complex variables, II

By

Tetsuo UEDA

This is the continuation of our previous paper of the same title [I]. We continue the investigation of semi-attractive and semi-repulsive transformations of type $(1, b)_1$.

Let us briefly recall some definitions and results in [I]. By an analytic transformation of two complex variables we mean (the germ of) a holomorphic mapping of a neighborhood of $O = (0, 0) \in \mathbf{C}^2$ into $\mathbf{C}^2$ such that $T(O) = O$. We say that T is of type $(1, b)$ if the eigenvalues of the linear part of T are 1 and b. When $b \neq 1$, we can choose local coordinates (x, y) around O so that $T: (x, y) \mapsto (x_1, y_1)$ takes the form

$$(0.1) \qquad \begin{cases} x_1 = x + \sum_{i+j \geqq 2} a_{ij} x^i y^j \\ y_1 = by + \sum_{i+j \geqq 2} b_{ij} x^i y^j . \end{cases}$$

In [I, Sec. 6] we showed that every transformation T of type $(1, b)$ with $|b| \neq 0, 1$ is equivalent to a transformation $(z, w) \mapsto (z_1, w_1)$ of a neighborhood of $(\infty, 0) \in \hat{\mathbf{C}} \times \mathbf{C}$ into $\hat{\mathbf{C}} \times \mathbf{C}$ of the form

$$(0.2) \qquad \begin{cases} z_1 = z + a_0 + \dfrac{a_1}{z} + \dfrac{a_2(w)}{z^2} + \cdots \\ w_1 = bw + \dfrac{b_1 w}{z} + \dfrac{b_2(w)}{z^2} + \cdots \end{cases}$$

where a_0, a_1, b_1 are constants and $a_j(w)$ $(i = 2, 3, \ldots)$, $b_j(w)$ $(j = 2, 3, \ldots)$ are holomorphic functions of one complex variable w in a neighborhood of $w = 0$. This transformation will be regarded as an expression of T with respect to the "local coordinate system" (z, w) around $O = (\infty, 0)$ and denoted also by T.

T is said to be of type $(1, b)_1$ if $a_0 \neq 0$ in the expression (0.2). This is equivalent to the condition $a_{20} \neq 0$ in (0.1).

In what follows, we assume that $0 < |b| < 1$, so that T is semi-attractive and its inverse T^{-1} is semi-repulsive. Further, for the simplicity of the argument, we assume that a_0 is a real positive number. The result for the general case

Received, Oct. 5, 1989