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On the asymptotic behavior of Gaussian sequences with stationary increments

By

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1. Introduction

Let $\{X(n); n = 1, 2, \dots\}$ be a stochastic process with discrete time parameter and introduce the following definitions.

Definition 1.1. The function $g(n)$ $(n = 1, 2, \dots)$ belongs to the upper-upper class of the process $X(n)$ $(g \in \mathrm{UUC}(X))$ if almost surely there exists $n_0 > 0$ such that for all $n \geq n_0$, $X(n) < g(n)$ holds.

Definition 1.2. The function $g(n)$ $(n = 1, 2, \dots)$ belongs to the upper-lower class of the process $X(n)$ $(g \in \mathrm{ULC}(X))$ if almost surely there exists an infinite sequence $0 < n_1 < n_2 < \cdots \to +\infty$ such that for all k, $g(n_k) \leq X(n_k)$ holds.

Recently, the second author [9] has investigated the asymptotic behavior of the increments of a Wiener process $W(t)$ $(0 \leq t < \infty)$ using the above notion of UUC and ULC obviously modified for processes with continuous time parameter. In order to compare our results with his, we briefly summarize main parts of his results: Let $\tilde{a}_T$ $(0 \leq T < \infty)$ be a real function of T satisfying the conditions

(i) $0 < \tilde{a}_T \leq T$,

(ii) $\tilde{a}_T$ is nondecreasing,

(iii) $T - \tilde{a}_T$ is nondecreasing.

Denote, for $0 \leq T < \infty$

$$X_0(T) = \sup_{0 \leq s \leq \tilde{a}_T} \sup_{0 \leq t \leq T-s} |W(s+t) - W(t)| / \sqrt{\tilde{a}_T}$$

$$X_1(T) = \sup_{0 \leq s \leq \tilde{a}_T} \sup_{0 \leq t \leq T-s} (W(s+t) - W(t)) / \sqrt{\tilde{a}_T}$$

$$X_2(T) = \sup_{0 \leq s \leq \tilde{a}_T} \sup_{0 \leq t \leq T-\tilde{a}_T} |W(s+t) - W(t)| / \sqrt{\tilde{a}_T}$$

$$X_3(T) = \sup_{0 \leq s \leq \tilde{a}_T} \sup_{0 \leq t \leq T-\tilde{a}_T} (W(s+t) - W(t)) / \sqrt{\tilde{a}_T}$$

$$X_4(T) = \sup_{0 \leq t \leq T-\tilde{a}_T} |W(t+\tilde{a}_T) - W(t)| / \sqrt{\tilde{a}_T}$$

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