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# On the composition of functions of bounded mean oscillation with meromorphic functions

Dedicated to Professor Tatsuo Fuji'i'e on his sixtieth birthday

By

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## Introduction

A quasi-conformal mapping preserves *BMO*, that is, if $g: \Omega_1 \to \Omega_2$ is a quasiconformal mapping between plane domains, then for every $f \in BMO(\Omega_2)$, $f \circ g$ belongs to $BMO(\Omega_1)$. In our former paper we partially extended this result by characterizing the analytic functions which preserve *BMO*. In this paper we treat more generally meromorphic functions. We shall characterize the Blaschke type meromorphic functions preserving *BMO* (Theorem 1).

## § 1. Main Theorem

Let $\Omega$ be a domain on complex plane **C**. $BMO(\Omega)$ is the space of all locally integrable functions $f$ on $\Omega$ such that

$$\|f\|_{*,\Omega} = \sup m(B)^{-1} \int_B |f - f_B|\, dm < \infty$$

where $dm$ is the 2-dimensional Lebesgue measure, $f_B$ is the integral mean of $f$ on $B$ and the supremum is taken for every disk $B$ in $\Omega$. $BMO(\mathbf{C})$ coinsides with the *BMO* space on the complex sphere $\hat{\mathbf{C}}$ with respect to its surface measure (cf. [10]), and $BMO(\mathbf{C})$ is obviously invariant under dilations and translations, especially it is invariant under Möbius transformations of $\hat{\mathbf{C}}$. More generally, Reimann and Jones proved the following result;

**Proposition 1** ([7], [9]). *Let $\Omega_1$ and $\Omega_2$ be plane domains and $g: \Omega_1 \to \Omega_2$ a quasi-conformal mapping then for every $f \in BMO(\Omega_2)$, $f \circ g$ belongs to $BMO(\Omega_1)$ and it holds that $\|f \circ g\|_{*,\Omega_1} \leq C \|f\|_{*,\Omega_2}$ where $C > 0$ is a constant depending only on the maximal dilatation of $g$. Conversely if $g$ is an absolutely continuous homeomorphism which preserves BMO then $g$ is a quasi-conformal mapping.*

In our former papers, we characterized the analytic function which preserves *BMO*, *BMOH*, and *BMOA* as follows, where *BMOH* (resp. *BMOA*) is the space of all harmonic (analytic) *BMO* functions;

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