J. Math. Kyoto Univ. (JMKYAZ)
31-4 (1991) 1023-1062

Theory of G-categories toward equivariant algebraic K-theory

By

Masahiko NIWA*

The notion of a G-category —a category with an action of a group G— was needed to make algebraic K-theory equivariant one. Though various notions have been used so far, the relations with them have not been explained explicitly yet. Beginning by introducing the notion of a G-category from point of view of Galois descent in linear categories, I deal comprehensively with various notions of G-categories and establish the comparison in the complete form. It is important for us to study simultaneously the limit categories together with G-categories and G-functors. The objects to appear in text are as follows.

G-category	G-functor	limit category
a category C with a G-descent datum	a morphism of Galois descent data	descended category $\Delta_H C$
a pseudo functor $\alpha: G \to Cat$	a pseudo nat. transf. $G \overset{\longrightarrow}{\underset{\longrightarrow}{\downarrow}} Cat$	
a fibered category over G $\gamma: D \to G$	a cartesian functor over G; $D \longrightarrow D'$ $\searrow_G \swarrow$	representation category $Cart_G(H, D)$ or $Cart_G(\underline{\underline{G/H}}, D)$
a lax functor	a lax nat. transf.	lax limit over G
a (strict) functor $\alpha: G \to Cat$	a nat. transf. $G \overset{\longrightarrow}{\underset{\longrightarrow}{\downarrow}} Cat$	$\Delta_H \alpha(\cdot)$ or $\alpha(\cdot)^H$
an $O_G{}^{op}$-category $\beta: O_G{}^{op} \to Cat$	a nat. transf. $O_G{}^{op} \overset{\longrightarrow}{\underset{\longrightarrow}{\downarrow}} Cat$	$\beta(G/H)$

§ 1. Introduction: The notion of G-categories

In order to introduce the notion of G-categories *i.e.* categories on which the group G acts, I think, we are asked to fit it to the following problems. One of them is the

* This research was partially supported by Grant-in-Aid for Scientific Research (C) No. 62540037
Communicated by Prof. H. Toda, Feb. 19, 1990