J. Math. Kyoto Univ. (JMKYAZ)
32-1 (1992) 57–72

On the norm of the Poincaré series operator for a universal covering group

Dedicated to Professor Tatuo Fuji'i'e on his sixtieth birthday

By

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Introduction

Let Δ be the unit disk and Γ be a Fuchsian group acting on Δ which may be elementary or not. All Fuchsian groups considered in this paper are assumed to be torsionfree, that is, they are covering groups of some universal coverings. We denote by $A(\Delta, \Gamma)$ the Banach space of all holomorphic functions ϕ on Δ with

$$\gamma^*\phi = \phi \qquad \text{for all } \gamma \in \Gamma,$$

where $\gamma^*\phi = (\phi \circ \gamma)(\gamma')^2$, and norm

$$\|\phi\|_\Gamma = \iint_{\Delta/\Gamma} |\phi(z)|\, dx\, dy < \infty.$$

When Γ is the trivial group $\{1\}$, we abbreviate $A(\Delta, \{1\})$ and $\|\phi\|_{\{1\}}$ by $A(\Delta)$ and $\|\phi\|$, respectively.

For Γ and its subgroup Γ_1, the Poincaré series operator $\Theta_{\Gamma_1\backslash\Gamma}: A(\Delta, \Gamma_1) \to A(\Delta, \Gamma)$ is defined by

$$\Theta_{\Gamma_1\backslash\Gamma}\phi = \sum_{\gamma \in \Gamma_1\backslash\Gamma} \gamma^*\phi.$$

When $\Gamma_1 = \{1\}$, we simply denote Θ_Γ for $\Theta_{\{1\}\backslash\Gamma}$. It is known that $\Theta_{\Gamma_1\backslash\Gamma}$ is an open continuous surjection with norm at most one and satisfies $\Theta_\Gamma = \Theta_{\Gamma_1\backslash\Gamma} \circ \Theta_{\Gamma_1}$ (cf. Kra [3, p.91]).

We have shown in [9]

Theorem A. *Let Γ be a Fuchsian group acting on Δ and Γ_1 be a normal subgroup of Γ such that $\Gamma_1\backslash\Gamma$ is finitely generated and abelian. Then, for all nonzero $\Phi \in A(\Delta, \Gamma)$, we have*

$$\sup\left\{\frac{\|\Phi\|_\Gamma}{\|\phi\|_{\Gamma_1}} : \Phi = \Theta_{\Gamma_1\backslash\Gamma}\phi,\ \phi \in A(\Delta, \Gamma_1)\right\} = 1,$$

Received May 28, 1990