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On the Bers conjecture for Fuchsian groups of the second kind

Dedicated to Proffessor Tatsuo Fuji'i'e on his sixtieth birthday

By

Toshiyuki SUGAWA

§ 1. Introduction

Suppose that D is a simply connected domain of hyperbolic type in the Riemann sphere $\hat{\mathbf{C}} = \mathbf{C} \cup \{\infty\}$. Then the Poincaré metric ρ_D in D is defined by

$$\rho_D(z) = \frac{|g'(z)|}{1 - |g(z)|^2}, \qquad z \in D,$$

where g is any conformal mapping of D onto the unit disk $\Delta = \{z : |z| < 1\}$. $B_2(D)$ will denote the Banach space consisting of all holomorphic functions φ in D such that the norm

$$\|\varphi\|_D = \sup_{z \in D} |\varphi(z)| \rho_D(z)^{-2}$$

is finite.

If f is a locally univalent meromorphic function, the Schwarzian derivative of f is given by

$$S_f = \left(\frac{f''}{f'}\right)' - \frac{1}{2}\left(\frac{f''}{f'}\right)^2.$$

We set after Flinn [6]

$$S = \{S_f : f \text{ is conformal in } \Delta\},$$
$$J = \{S_f \in S : f(\Delta) \text{ is a Jordan domain}\},$$
$$T = \{S_f \in S : f(\Delta) \text{ is a quasidisk}\}.$$

T is called the universal Teichmüller space. It is known that $T \subset J \subset S \subset B_2(\Delta)$, T is open, S is closed and $T = \operatorname{Int} S$ (see [7], [9]). Let Γ be a Fuchsian group acting on Δ and $B_2(\Delta, \Gamma)$ denote the closed subspace of $B_2(\Delta)$:

$$\{\varphi \in B_2(\Delta) : (\varphi \circ \gamma) \cdot (\gamma')^2 = \varphi \qquad \text{for all } \gamma \in \Gamma\}.$$

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