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Semi-classical asymptotics for total scattering cross sections of 3-body systems

Dedicated to Professor Teruo Ikebe on his sixtieth birthday

By

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Introduction

The scattering cross section is directly related to experimental observations in laboratories and is one of the most important quantities in scattering theory. There are many works on the semi-classical analysis for scattering matrices of 2-body systems. For example, such a problem has been studied for scattering amplitudes in the works [14, 18, 21] and for total scattering cross sections in the works [7, 13, 16, 19, 20]. In the present work, we study the semi-classical asymptotic behavior of total scattering cross sections with 2-body initial states for 3-body systems. Such an initial state is of most practical interest. In fact, for many-body scattering systems, k-body initial states with $k \geq 3$ are not easy to realize through actual physical experiments. There seems to be only a few works on the analysis for scattering matrices of many-body systems. In a series of works [3, 4, 5], the following properties of total scattering cross sections have been studied in detail: (1) finiteness of total scattering cross sections; (2) continuity as a function of energy; (3) behavior at high and low energies. The asymptotic behavior in the semi-classical limit has not been discussed in detail in these works.

Throughout the entire discussion, the constant h, $0 < h \ll 1$, denotes a small parameter corresponding to the Planck constant. We require several basic notations and definitions in many-body scattering theory to define precisely the total scattering cross section in question. We here state our main theorem somewhat loosely. The precise formulation of the main result is given as Theorem 1.1 in section 1.

Consider a system consisting of three particles moving in the 3-dimensional space R^3 through real pair potentials V_{jk}, $1 \leq j < k \leq 3$. We denote by m_j, $1 \leq j \leq 3$, the mass of the j-th particle and by $r_j \in R^3$ its position vector. A partition of $\{1, 2, 3\}$ into nonempty disjoint subsets is called a cluster decomposition. We use the letter a or b to denote such a cluster decomposition. The Jacobi coordinates $(y_a, z_a) \in R^{3 \times 2}$ associated with given 2-cluster decomposition $a = \{l, (j, k)\}$ with $j < k$ are defined as

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