J. Math. Kyoto Univ. (JMKYAZ)
32-3 (1992) 451-459

On homotopy associative mod 2 H-spaces[1]

By

John McCleary

In [1], [3], and [6] the following question is considered:

If Y is a mod 2 H-space, when does $Y \times S^7$ admit the structure of a homotopy associative mod 2 H-space?

Among the simple Lie groups, the results in [1] reveal that the only possible examples are the following:

$$Spin(7)_{(2)} \simeq (G_2 \times S^7)_{(2)}$$

$$Spin(8)_{(2)} \simeq (Spin(7) \times S^7)_{(2)}$$

$$\text{and } SO(8)_{(2)} \simeq (SO(7) \times S^7)_{(2)}\,.$$

The focus of [6] is on generalizing the results of [1] to finite H-spaces. Here the Hopf algebra over the mod 2 Steenrod algebra, $\mathscr{A}_2$, given by

$$A = F_2[x_3]/x_3^4 \otimes \Lambda(Sq^2 x_3) \cong H^*(G_2; F_2)$$

plays a crucial role. The main results of [6] are summarized in the following

Theorem (Lin-Williams). *Let Y be a finite simply-connected CW-complex and suppose $H^*(Y; F_2)$ contains no subalgebras isomorphic to A. Then $Y \times S^7$ cannot be a homotopy associative H-space. Suppose $H^*(Y; F_2)$ contains at most one subalgebra isomorphic to A. Then $Y \times (S^7)^k$ cannot be a homotopy associative H-space for $k \geq 3$.*

The method of proof of this theorem suggests an extension that is the principal result of this paper.

Main Theorem. *Let Y be a finite simply-connected CW-complex and suppose that $H^*(Y; F_2)$ contains $A^{\otimes \ell}$ for some $\ell \geq 0$. Then $Y \times (S^7)^k$ cannot be a homotopy associative H-space for $k \geq 2\ell + 1$.*

This shows that, at the prime 2, from ℓ copies of G_2 and k copies of S^7, the examples of $Spin(7)$ and $Spin(8)$ above are the only homotopy associative

Communicated by Prof. K. Ueno, June 8, 1990

[1] This paper was written while I was visiting the Mathematical Sciences Research Institute, Berkeley, CA.